

Study Guide for Test # 2

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MATH 1113

Sections 4.5–4.7, 5.1–5.7

Show your work clearly to get credit.

1. Polynomial and Rational Inequalities.

(a) Solve $x^3 + 2x^2 - 3x > 0$.

(b) Solve $\frac{5}{x-3} > \frac{3}{x+1}$.

2. Real Zeros of a Polynomial Function.

(a) Use Descartes' Rule of Signs and the Rational Zeros Theorem to find all the real zeros of $f(x) = x^3 + 8x^2 + 11x - 20$. Use the zeros to factor $f(x)$ over the real numbers.

(b) Find the intercepts of $f(x) = 4x^5 + 12x^4 - x - 3$. Find the intervals for which the graph of $f(x)$ is above the x -axis and below the x -axis. Sketch the curve $y = f(x)$.

(c) Find a bound on the real zeros of $f(x) = x^4 - 5x^2 - 36$.

(d) Show that the function $h(x) = x^4 + 8x^3 - x^2 + 2$ has a zero on $[-1, 0]$ and approximate this zero correct to two decimal places.

3. Complex Zeros of a Polynomial Function.

(a) Form a fourth degree polynomial $f(x)$ with real coefficients having zeros $1 - 2i$ and 2 of multiplicity 2.

(b) Use a zero $3i$ of $g(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$ to find the remaining zeros.

(c) Factor the polynomial $f(x) = x^4 + 3x^3 - 19x^2 + 27x - 252$ completely.

4. Composite Functions.

(a) For $f(x) = \frac{2}{x+1}$ and $g(x) = \frac{3}{x}$, find $f \circ g$ and $g \circ f$. State the domain of each composite function.

(b) Find $f(x)$ and $g(x)$ such that $(f \circ g)(x) = (x^2 + 2x - 1)^3$.

5. Inverse Functions.

(a) Find the inverse of $f(x) = \frac{3x}{2x+1}$. State the domain of f and find its range using f^{-1} .

(b) Use the Horizontal Line Test to show that the function $f(x) = x^2 + 4x - 3$, $x \geq -2$, is one-to-one.

(c) Find the inverse function to the function $f(x)$ in (b).

(d) Sketch the graph of the inverse function to the function $f(x)$ in (b) using the graph of $f(x)$.

6. Exponential Functions.

(a) Use transformations to graph $f(x) = 2(4^{-x}) - 3$. Determine the domain, range, and the horizontal asymptote.

(b) Solve the equation $2^{x^3-x} = 8^x$.

(c) Solve the equation $16^x - 4^x - 12 = 0$.

7. Logarithmic Functions.

(a) Find the domain of $f(x) = \ln\left(\frac{x+1}{x-2}\right)$.

(b) Use the graph of a basic logarithmic function and graph transforms to sketch the graph of $g(x) = 3\log_{1/2}(x+2) - 1$.

(c) Solve the equation $\log_x\left(\frac{1}{16}\right) = -4$.

(d) Solve the equation $\log_4(x-3) + \log_4 x = 1$.

(e) Find $3^{2\log_3 2 - \log_3 5}$ using properties of logarithmic functions.

(f) If $\ln x = 2$, $\ln y = -3$, and $\ln z = 4$, find the exact value of $\ln\left(\frac{\sqrt{xy^3}}{z^4}\right)$ using algebraic properties of logarithmic functions.

(g) Use the change of base formula to compute $A = \log_2 3 \cdot \log_3 5 \cdot \log_5 8$.

8. Compound interest.

(a) \$5000 invested at 6% compounded monthly. Find the amount of this investment after a period of 5 years.

(b) Find the present value needed to get \$1000 after 4 years at 5% compounded continuously.

(c) What rate of investment compounded annually is required to double an investment in 10 years?