Math 1113 **Practice Test #2** **Name**_____
F. Ziegler **Fall 2004**

You must show your work to receive credit. Use proper mathematical sentences (= signs, parentheses, etc.). If you depend on the calculator for an answer, explain exactly what buttons you pushed. **Be sure to spell out a clear answer to every question asked.**

1. Solve the equations. (Give exact formulas for all real solutions.)

a) $3^{1+2x} = 27$

b) $4^x = 16$

c) $4^{x+4} = 5^{2x+5}$

d) $\log_3 x + \log_3(x - 24) = 4$

2. a) Write as a single logarithm: $6\log(p) + \log(q) - \frac{1}{2}\log(r - 1)$

b) Write as a sum or difference of logarithms, expressing powers as factors: $\ln\left(\dfrac{(x+9)(x-2)}{(x-3)^2}\right)^{5/2}$

3. A sum of \$1000 is invested at an annual interest rate of 10%. How much is it worth, to the nearest cent, after 100 years if compounded

a) Yearly?

b) Monthly?

c) Continuously?

4. The formula $v = 120(1-e^{kt})$, where k is a negative constant, gives the downward speed in mph of a skydiver t seconds after jumping.

a) What is the speed initially (at $t = 0$)?

b) What is the speed ultimately (asymptote as $t \to \infty$)?

c) v is measured to be 84 mph after 6 seconds. What is k? (Give an exact formula.)

d) What will the speed be after 18 seconds? (Compute to the nearest 0.1 mph.)

5. The number 1988 is four digits long. How many digits long is the number 3^{100} (expressed in base 10)?

6. At noon in early summer, the sun's rays fall exactly vertically on the city of Aswan, Egypt. Meanwhile in Alexandria, which is at a distance of 5000 *stadia* due North, the rays make a $7°12'$ angle with the vertical. Using these observations, the Greeks got the first estimate of the Earth's circumference. What value did they obtain, in *stadia*?

7. A right triangle has perpendicular sides of lengths 7 miles and 24 miles. Sketch the triangle, name and clearly label its 3 angles. Then compute the exact value of the sine, cosine and cotangent of each angle.

8. The formula $A = \frac{1}{2} d_1 d_2 \sin\theta$ gives the area of a quadrilateral whose diagonals have lengths d_1, d_2 and intersect at an angle θ. Give exact values for A when $\theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.

9. In each of the following, find an acute (first quadrant) angle which has the same sine up to a $\pm$ sign, and determine the sign. Express you answer as in a).

a) $\sin(135°) = +\sin(45°)$

b) $\sin(-15^\circ) =$

c) $\sin(\frac{33\pi}{7}) =$

d) $\sin(888^\circ) =$

10. Give the period, phase shift and (if appropriate) amplitude of the following functions. Use transformations to sketch the graph of each. Be sure to show at least one full period, and label the axes to identify the position of extrema, intercepts and asymptotes.

a) $f(x) = -3\cos(x - \frac{\pi}{4})$

b) $g(x) = 1 + \tan(3x + \frac{\pi}{2})$