

Show your work to receive credit! Use proper mathematical sentences (= signs, parentheses, etc.), and **be sure to spell out a clear answer to every question asked**. If you depend on the calculator for an answer, explain exactly what buttons you pushed.

1. Find all real solutions of the equations:

a) $6x^2 - 5x - 14 = 0$.

b) $x^{-6} - 7x^{-3} - 8 = 0$.

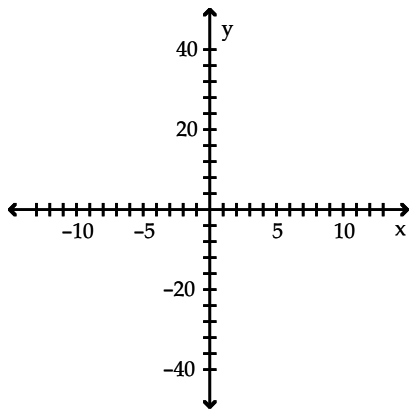
c) $\sqrt{2x - 1} = x - 2$.

d) $3^{6x^2+2x} = \sqrt{3}$.

e) $2^x \cdot 5 = 10^x$.

f) $\log(x + 5) - \log(x - 5) = \log 3$.

2. Graph the quadratic function $f(x) = 12x - x^2$ using its vertex, axis of symmetry and intercepts. Label the vertex and intercepts by their coordinates (x, y) in the plane.



3. Consider the polynomial $f(x) = (2x + 3)(x + 1)^2(x - 1)^2(2x - 3)$.

a) Like what monomial does $f(x)$ behave for large values of $|x|$?

b) List all x -intercepts of f in increasing order.

c) Sign study: find in which intervals of the x -axis $f(x)$ is positive (resp. negative).

$f(x)$					

d) Sketch the graph of f . Can you identify any local maxima?

4. For each of the following functions, compute the difference of degree between numerator and denominator. Deduce whether the end behavior is a *horizontal asymptote*, an *oblique asymptote*, or *like a monomial*. (Further details are not required.)

a) $f(x) = \frac{2x^3 - 3x^2}{x^4 - x^3};$

degree(numerator) – degree(denominator) =

End behavior:

b) $f(x) = \frac{4x^5 - 5x^4}{x^4 - x^3}$

degree(numerator) – degree(denominator) =

End behavior:

c) $f(x) = \frac{6x^7 - 7x^6}{x^4 - x^3}$

degree(numerator) – degree(denominator) =

End behavior:

5. Use long division to find the equation of the asymptote of $f(x) = \frac{3x^3 + 10x^2 + 1}{x^2 + x + 1}$.

6. Consider the function $f(x) = \frac{(x^2 + 4)(x - 1)}{x^3}$
 $= 1 - \frac{(x - 2)^2}{x^3}$.

a) Find the domain of f , and all asymptotes of the graph.

b) Find all x - and y -intercepts.

c) Find any point (x, y) where the graph touches or crosses an asymptote.

d) Sign study: find in which intervals of the x -axis $f(x)$ is positive (resp. negative).

$f(x)$			

e) Sketch the graph of f . Display and label all elements found under a)–c).

7. a) Find any *rational* zeros of $f(x) = x^3 - 6x - 4$.

b) Now factor $f(x)$ (using long division if necessary) and find *all* its zeros.

8. The U.S. Treasury borrows \$100,000,000 at annual rate 3%. How much does it owe after 10 years, to the nearest dollar, if the interest is compounded

a) Yearly?

b) Weekly?

c) Continuously?

9. Solve for values of x in the interval $[0, 2\pi)$:

a) $\cos(3x) = 0$.

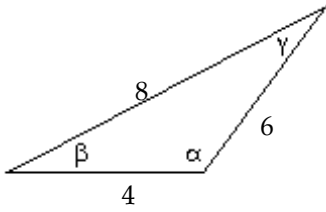
b) $\sin(2x) = \sin(x)$.

12. It is known that $\cos 36^\circ = \sqrt{\frac{3 + \sqrt{5}}{8}}$ and $\sin 36^\circ = \sqrt{\frac{5 - \sqrt{5}}{8}}$.

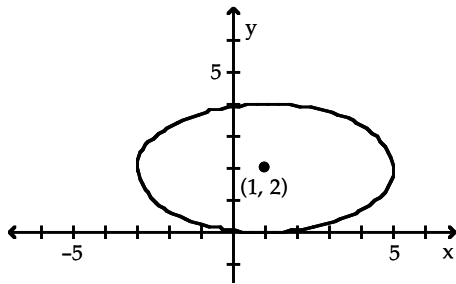
a) Deduce exact formulas for $\cos 72^\circ$ and $\sin 72^\circ$.

b) Compute $(\cos 36^\circ + i \sin 36^\circ)^5$. (Hint: *avoid* expanding the fifth power.)

11. Solve the triangle. Compute the angles to within 0.1° .



12. a) Write an equation for the ellipse:



b) What are the coordinates of its foci?

13. Compute the product $\begin{pmatrix} 5 & 1 & 6 \\ 1 & 5 & 6 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.

14. Solve the following system by performing elementary row operations on an augmented matrix. Show all stages and verify your answer.

$$\begin{cases} x - y + z = 0 \\ 2x - 2y + z = 1 \\ x - y - 5z = 6 \end{cases}$$

15. Invert the matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ by performing elementary row operations on an augmented matrix. Show all stages and verify your answer.