Mathematics Competition, Fall 2004, Georgia Southern University
Do any or all of these problems.
Due date: November 1, 2004 by 5:00 pm

Problem 1. A car holds 6 people (including the driver), 3 in the front seat and 3 in the back seat. How many different seating arrangements of the 6 people are possible if one person refuses to sit in the front and two different people (different from each other and different from the first) refuse to sit in the back? Assume all 6 are licensed drivers!

Problem 2. Let $x_0, \dots, x_n$ be real numbers. Show that

$$\det \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & & & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix} = \prod_{i>j}^{n} (x_i - x_j).$$

Problem 3. We say that x is a *fixed point* of f if $f(x) = x$. Show that:
(a) A continuous function $f : [0,1] \to [0,1]$ has a fixed point.
(b) A differential function on $(-\infty, \infty)$ with $f'(x) \neq 1$ for all x has at most one fixed point.

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