

Spring 2006 MPC – Set 1, Due by 5pm on Friday, Feb. 17.

Instructions: All GSU Undergrads are eligible. Submit your solutions to the Mathematics office. Please include your name and e-mail address. Have fun!

http://cost.georgiasouthern.edu/math/math_competition/math_competition.php

1. (G. Lesaja) Find all real functions of one real variable that satisfy the following functional equation

$$f(x) + 2f\left(\frac{1}{x}\right) = x.$$

2. (V. Maymeskul) Solve the following nonlinear system of equations (for ALL possible solutions):

$$\begin{aligned}a + b + c &= 7 \\ a^2 + b^2 + c^2 &= 21 \\ abc &= 8\end{aligned}$$

Hints: Note that $x = a, b$ and c are the roots of the cubic $f(x) = (x - a)(x - b)(x - c)$. Also consider $(a + b + c)^2$.

3. (S. Kersey) Prove that $\int_{\pi/4}^{\pi/2} \frac{\sin(x)}{x} dx < \frac{\sqrt{2}}{2}$. (Justify all steps.)