

Fall 2005 GSU Mathematics Problem Competition – Problem Set 2

Due Date: Monday, Oct. 10 by 5p.m.

Instructions: Submit your solutions to the Mathematics office or to MP 3004 by the due date. Please include your name and e-mail address. Have fun! If you have any questions please contact skersey@GeorgiaSouthern.edu. See solutions to Set 1 at:

http://www.cs.georgiasouthern.edu/faculty/kersey_s/private/competition/competitionS2005.html

1. (F. Ziegler) Prove that for all non-negative real numbers a and b

$$|\sqrt{a} - \sqrt{b}| \leq \sqrt{|a - b|}.$$

2. (B. Oluyede) Let $k > 1$. Suppose that $\sum_{i=1}^k p_i = 1 = \sum_{i=1}^k q_i$ for some $p_i, q_i \geq 0$, and that

$$\left(\sum_{i=1}^j p_i\right)q_{j+1} \geq \left(\sum_{i=1}^j q_i\right)p_{j+1} \quad \text{for } 1 \leq j \leq k-1.$$

- a) Give an example (choose p_i, q_i) where this last condition fails for $k = 3$.
b) Show that

$$\sum_{i=1}^j p_i \geq \sum_{i=1}^j q_i \quad \text{for } 1 \leq j \leq k.$$