Fall 2006 MPC – Set 2, Due by 5pm on Friday, **Oct 27**.
Instructions: All GSU Undergrads are eligible. Submit your solutions to the Mathematics office. Please include your name and e-mail address. Have fun!
http://cost.georgiasouthern.edu/math/math_competition/math_competition.php

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1. Construct a 2×2 matrix A with distinct entries chosen from $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6\}$ so that $A^2 = A$ (such a matrix is called idempotent). Can you find all of them?

2. Let $A = (a_{ij})_{n \times n}$ be a $n \times n$ matrix with $a_{ij} = 1$ for all i and j, i.e., all entries of A are 1's, and let I_n denote the unit $n \times n$ matrix. Show that the zeros of the characteristic polynomial $\chi(\lambda) = \det(A - \lambda I_n)$ are $\lambda = 0$ of multiplicity $n - 1$ and $\lambda = n$. (Zeros of the characteristic polynomial are called the eigenvalues of A).

3. Given that a polynomial $P(x) = x^{20} - 20x^{19} + \boxed{\text{“\textit{black box}”}} + 1$ has all real positive roots, recover the polynomial and find its roots.