Spring 2005 Mathematics Problem Competition — Georgia Southern University
Problem Set 2 — Due Date: Friday, February 25, 2005

Instructions: Do any or all of these problems, and submit to the Mathematics main office by the due date, or give to your instructor. All GSU undergraduate students are eligible. Please include your name and e-mail address or math instructor's name. Have fun!

---

1. Recall by the Fundamental Theorem of Algebra that a polynomial of degree n has n (real or complex) roots. Factor $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$ completely (by hand).

2. Suppose that x, y and z are non-zero numbers such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{x+y+z}$. Show that $\frac{1}{x^n} + \frac{1}{y^n} + \frac{1}{z^n} = \frac{1}{(x+y+z)^n}$ where n is an odd integer. (Hint: Start by solving the first equation for z in terms of x and y.)

3. Suppose that $x_1, x_2, x_3, \ldots$ is a sequence of real numbers such that $|x_{n+1} - p| < \frac{1}{2}|x_n - p|$ for all n. Show that x_n converges to p.

4. Suppose you are dealt a 5 card poker hand from a standard deck of cards. Is a flush (5 cards in the same suit) or a full house (e.g., 33322) a better hand? Support your claim.

5. Professor Stone says he has a magic money-making machine. You start by giving him \$x. If x is an odd number, the machine will triple your money plus add one more dollar, and put the result back in the machine to make more money; or if x is even, the machine will halve the amount of your money, and again put your money back in the machine. For example, starting with x=\$27, your money grows as follows:

$$27,\ 3(27)+1 = 82,\quad \frac{82}{2} = 41,\quad 3*41+1 = 124,\quad \frac{124}{2} = 62,\quad \frac{62}{2} = 31,\quad 3*31+1 = 94,$$
$$\frac{94}{2} = 47,\quad 3*47+1 = 142,\quad \frac{142}{2} = 71,\quad 3*71+1 = 214,\quad \frac{214}{2} = 107,\quad 3*157+1 = 322, \ldots$$

Already your money has grown to \$322! Dr. Stone says he will run his machine for a "long time" so that you earn alot of money. Should you take his offer? How much are you willing to "invest"?