

Instructions: Do any or all of these problems, and submit to the Mathematics main office by 5:00 pm on the Due date. All GSU undergraduate students are eligible. Have fun!

Problem 1. Simplify $\sqrt{3 + 2\sqrt{2}}$.

Problem 2. Find all solutions for the equation $\sin(x) = x^2 + x + 1$. Justify your answer.

Problem 3. Let a and b be positive real numbers. Prove that $\frac{1}{(\frac{1}{a} + \frac{1}{b})/2} \leq \sqrt{ab} \leq \frac{a+b}{2}$.

That is, the *harmonic mean* \leq the *geometric mean* \leq the *arithmetic mean*.

Problem 4. Five balls are randomly distributed **inside** a square with unit length for each side. Prove that there are at least two balls, the distance between which is less than $\frac{1}{\sqrt{2}}$.

(Note: we assume the size of each ball is so small that it can be taken as a point in the square.)