Spring 2005 Mathematics Problem Competition — Georgia Southern University
Problem Set 3 — Due Date: Tuesday, March 29, 2005

Instructions: Solve as many problems as you can. Submit your solutions to the Mathematics main office by the due date, or give to your instructor. All GSU undergraduate students are eligible. Please include your name, e-mail address or math instructor's name. Have fun!
http://www.cs.georgiasouthern.edu/faculty/kersey_s/private/competition/competitionS2005.html

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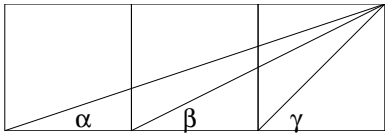
1. Show that if, in a forest, the number of trees is greater than the number of leaves on any of them and there are no bare trees, then at least two trees have the same number of leaves.

2. In the picture three squares are side by side, and α, β and γ are angles. Prove that $\alpha + \beta = \gamma$.



3. Can a square matrix A with integer elements and $\det(A) = 1$ have eigenvalue $1/2$? (Hints: Characteristic polynomial, Rational Root Theorem.)

4. Suppose that $|f(x) - f(y)| \leq M|x - y|^{\alpha}$ for all x and y, with $M > 0$, $\alpha > 0$. Prove that:
   (a) f is continuous.
   (b) If $\alpha > 1$ then f is a constant function.
   (c) If $\alpha = 1$ and f is differentiable, then $|f'(x)| \leq M$.
   (d) If f is differentiable and $|f'(x)| \leq M$, then $\alpha = 1$ holds true above.