

Spring 2006 MPC – Set 3, Due by 5pm on Friday, April 28.

**Instructions:** All GSU Undergrads are eligible. Submit your solutions to the Mathematics office. Please include your name and e-mail address. Have fun!

[http://cost.georgiasouthern.edu/math/math\\_competition/math\\_competition.php](http://cost.georgiasouthern.edu/math/math_competition/math_competition.php)

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1. In how many ways a positive integer  $n$  can be represented as a sum of positive integers if the order matters (i.e.,  $1 + 2 + 3$  is different from  $1 + 3 + 2$ )?

2. If, in a right triangle, lengths of all sides are integers, show that the length of at least one leg is divisible by 3.

3. Show that  $\cos 5^\circ + \cos 77^\circ + \cos 149^\circ + \cos 221^\circ + \cos 293^\circ = 0$ .

(Moreover, the following more general result is true: for any real  $\alpha$  and integer  $n \geq 2$ ,  $\sum_{k=0}^{n-1} \cos(\alpha + 360^\circ k/n) = 0$ .)

4.  $N$  cups are originally in the upside down position. You want them to be in the right position, but you are allowed to turn over *exactly*  $N - 1$  cups at once. Show that the problem is solvable if and only if  $N$  is even, and find a solution.