Pontryagin duality and number theory

Gábor Lukács

February 22, 2008, Georgia Southern University, Statesboro, GA, USA

ABSTRACT

Let $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ denote the circle group, written additively, and for a sequence $\underline{u} = (u_n)$ of integers, put $s_{\underline{u}}(\mathbb{T}) = \{x \in \mathbb{T} \mid u_n x \longrightarrow 0\}$. Bíró, Deshouillers, and Sós proved that for every countable subgroup $H \leq \mathbb{T}$, there is a sequence $\underline{u}$ of integers such that $H = s_{\underline{u}}(\mathbb{T})$ (cf. [2, Theorem 2]). Although this result is number theoretic in nature, it turns out to be intimately related to topological groups. The *Pontryagin dual* of an abelian topological group G is the group $\hat{G}$ of continuous homomorphisms $\chi\colon G \to \mathbb{T}$, equipped with the compact-open topology. For instance, for $G = \mathbb{T}$, one has $\hat{G} = \mathbb{Z}$. In this talk, we present a survey of the Pontryagin duality, and show how it can be used to prove a far reaching generalization of the result of Bíró, Deshouillers, and Sós.

REFERENCES

[1] M. Beiglböck, C. Steineder, and R. Winkler. Sequences and filters of characters characterizing subgroups of compact abelian groups. *Topology Appl.*, 153(11):1682–1695, 2006.

[2] A. Bíró, J.-M. Deshouillers, and V. T. Sós. Good approximation and characterization of subgroups of $\mathbb{R}/\mathbb{Z}$. *Studia Sci. Math. Hungar.*, 38:97–113, 2001.

[3] Dikran Dikranjan. Closure operators in topological groups related to von Neumann's kernel. *Topology Appl.*, 153(11):1930–1955, 2006.

[4] Dikran Dikranjan and Kenneth Kunen. Characterizing subgroups of compact abelian groups. *J. Pure Appl. Algebra*, 208(1):285–291, 2007.

[5] Dikran Dikranjan, Chiara Milan, and Alberto Tonolo. A characterization of the maximally almost periodic abelian groups. *J. Pure Appl. Algebra*, 197(1-3):23–41, 2005.

Department of Mathematics
University of Manitoba
Winnipeg, R3T 4X2, Manitoba
Canada

e-mail: lukacs@cc.umanitoba.ca