Triquotient maps via ultrafilter convergence

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Our interest in ultrafilter convergence was motivated by the work of G. Janelidze and M. Sobral [3] on (Grothendieck) descent theory in finite topological spaces, where, according to them, descriptions of various kind of descent maps "become very simple and natural as soon as they are expressed in the language of finite preorders". The preorder relation of a (finite) space is just the point convergence shadow of the ultrafilter convergence relation, hence one might expect to obtain results about arbitrary spaces by "just" replacing point convergence by ultrafilter convergence. Following this idea, we will present characterisations of the classes of open maps, proper maps, triquotient maps and local homeomorphisms in terms of lifting properties of chains of convergent ultrafilters [1, 2]. To give an example, triquotient maps were introduced by E. Michael as those continuous maps $f : X \to Y$ for which there exists a mapping $(\_)^\sharp : \mathcal{O}X \to \mathcal{O}Y$ between the frames of open subsets which satisfies certain conditions. For finite spaces X and Y, f is a triquotient map if and only if each chain $y_n \to \cdots \to y_0$ of convergent points in Y is the image of a chain $x_n \to \cdots \to x_0$ of convergent points in X. For arbitrary spaces, we show that f is triquotient if and only if each (possibly infinite) chain $\cdots \mathfrak{Y}_2 \to \mathfrak{y}_1 \to y$ in Y of higher order ultrafilters is the image of a chain $\cdots \mathfrak{X}_2 \to \mathfrak{x}_1 \to x$ in X.

References

[1] M. M. Clementino and D. Hofmann, *Triquotient maps via ultrafilter convergence*, Proc. Amer. Math. Soc., 130 (2002), pp. 3423–3431.

[2] M. M. Clementino, D. Hofmann, and G. Janelidze, *Local homeomorphisms via ultrafilter convergence*, Proc. Amer. Math. Soc., 133 (2005), pp. 917–922.

[3] G. Janelidze and M. Sobral, *Finite preorders and topological descent. I*, J. Pure Appl. Algebra, 175 (2002), pp. 187–205. Special volume celebrating the 70th birthday of Professor Max Kelly.