

**GEORGIA SOUTHERN UNIVERSITY
INVITATIONAL MATHEMATICS TOURNAMENT
2002 VARSITY WRITTEN EXAM**

Name_____

School_____

Directions:

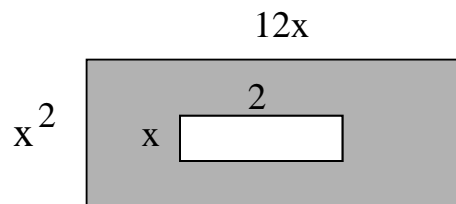
1. Do not open this test booklet until you are told to do so.
2. Use only a #2 lead pencil.
3. No calculators, slide rules, notes or other aids of any kind may be used.
4. Scratch paper is stapled to the back of the test booklet.
5. This is a 40 question multiple-choice exam. You will be allotted 90 minutes to complete the exam.
6. Geometric figures are not necessarily drawn to scale.
7. Your score will be determined by the formula $40 + 4R - W$ where R = number of questions answered correctly and W = number of questions answered wrong. There is no penalty for questions left unanswered.
8. Tie-breakers will be taken from the written exam in order of difficulty. This order will be determined by the number of people that answered each question correctly, with the question(s) correctly answered by the fewest people considered first.

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1. If $5x - 7 = 2x + 2$, evaluate $7x + 4$.

- A. -1 B. 25 C. 3
D. $-\frac{5}{7}$ E. none of the above

2. Write an expression for the area of the shaded region.



- A. $2x(6x^2 - 1)$ B. $2x^2 + 22x - 4$ C. $2x(6x^2 + 1)$
D. $2x^2 + 22x + 4$ E. none of the above

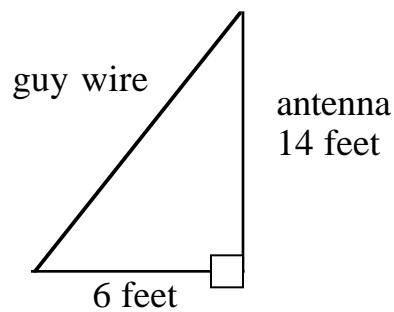
3. Evaluate $\left(\cos \frac{\pi}{3}\right)\left(\sin \frac{3\pi}{2}\right)$.

- A. $\frac{\sqrt{3}}{2}$ B. $\frac{1}{2}$ C. $-\frac{1}{2}$
D. $-\frac{\sqrt{3}}{2}$ E. none of the above

4. You have three separate large boxes, and inside each box are two separate medium boxes, and inside each of these medium boxes are four very small boxes. What is the total number of boxes?

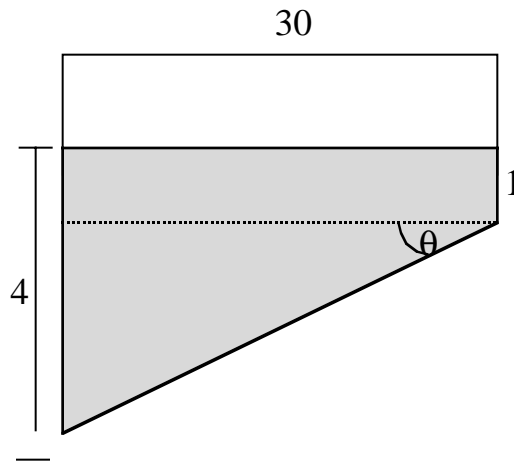
- A. 9 B. 33 C. 30
D. 21 E. none of the above

5. Roberto drew the adjacent sketch of the antenna and a guy wire support in his back yard. The antenna is 14 feet tall and the distance along the ground from where the guy wire is anchored to the base of the antenna is 6 feet. What is the slope of the guy wire?

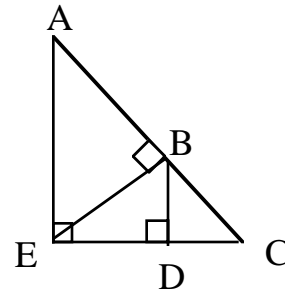


- A. $2\sqrt{58}$ B. $\frac{3}{7}$ C. 21
- D. $\frac{7}{3}$ E. none of the above
6. A equilateral triangle and a regular convex hexagon both have 12 inch perimeters. Let x = the area of the hexagon and y = the area of the triangle. Find $10\left(\frac{x}{y}\right)$.
- A. 30 B. $\frac{3}{2}$ C. 60
- D. 15 E. none of the above
7. Find the value of x in base ten if $x + 11_{\text{two}} = 25_{\text{ten}}$.
- A. 3 B. 13 C. 228
- D. 14 E. none of the above
8. Find the value of k such that the equations $(x - 2)^4 - (x - 2) = 0$ and $x^2 - kx + k = 0$ have two roots in common.
- A. 2 B. 4 C. -3
- D. 3 E. none of the above

9. A swimming pool is 30 meters long and 12 meters wide. The bottom of the pool is slanted so that the water depth is 1 meter at the shallow end and 4 meters at the deep end. (The water is shaded gray.) Find the angle of depression, θ , of the bottom of the pool.

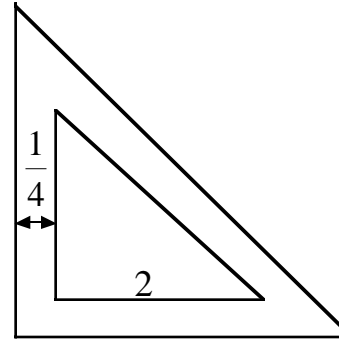


- A. $\tan^{-1}\left(\frac{2}{15}\right)$ B. $\tan^{-1}(10)$ C. $\tan^{-1}\left(\frac{15}{2}\right)$
 D. $\tan^{-1}\left(\frac{1}{10}\right)$ E. none of the above
10. Find the exact value of BD , given $AB = 24$ and $BC = 8$.



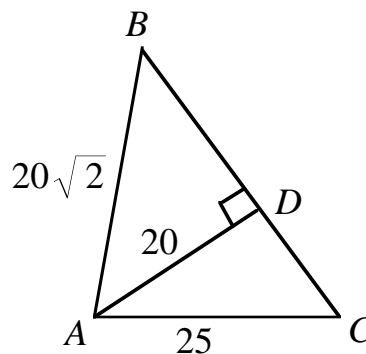
- A. $4\sqrt{3}$ B. 8 C. 4
 D. $2\sqrt{2}$ E. none of the above
11. A wheel's radius is $\frac{75}{\pi}$ inches, and it revolves 72° per second. Determine its linear velocity in inches per second.
- A. 30 B. $\frac{25}{24\pi}$ C. 60
 D. 15 E. none of the above

19. A quilter is cutting pieces that are congruent isosceles right triangles. When the quilt is finished, the part of each triangle that will show will have legs 2 inches long. The quilter is including a quarter-inch seam allowance on all three sides of the triangle to allow for piecing. How long should the quilter make each leg of the triangle?



- A. $\frac{10 + \sqrt{2}}{4}$ B. $\frac{9 + \sqrt{2}}{4}$ C. $\frac{11}{4}$
 D. $\frac{9 + 2\sqrt{2}}{4}$ E. none of the above

20. Find the altitude of triangle ABC from B to the base AC.



- A. 20 B. $5\sqrt{33}$ C. 28
 D. $14\frac{2}{7}$ E. none of the above

21. Simplify the following: $(2x^3)(5x)^{-2}$

- A. $\frac{x^2}{25}$ B. $\frac{2x}{25}$ C. $10x$
 D. $(2x)(5x)^{-1}$ E. none of the above

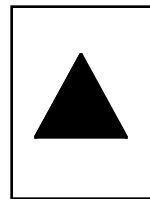
22. Suppose $\left(\frac{5}{13}, -\frac{12}{13}\right)$ is a point on the unit circle corresponding to the real number t , find $\tan t$.

- A. $\frac{12}{5}$ B. $-\frac{12}{5}$ C. $\frac{5}{12}$
 D. $-\frac{5}{12}$ E. none of the above

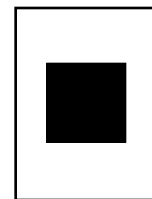
23. Turning over the fewest number of cards, which card(s) must one turn over to determine if every card with a triangle on one side is black on the other side?



1



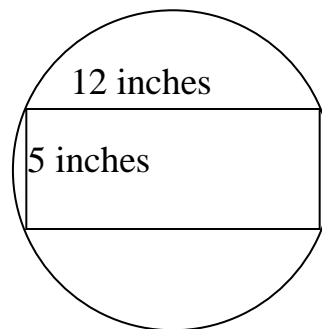
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3

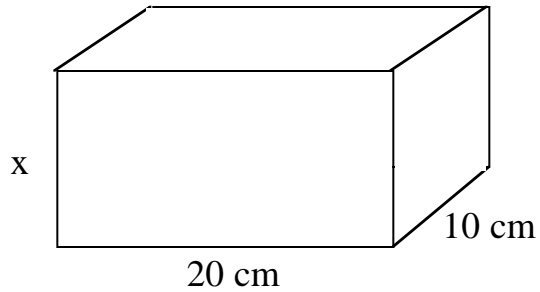
- A. 1 and 2 B. 2 and 3 C. 1
 D. 2 E. 1, 2, and 3

24. Find the radius of a circle that is circumscribed about a rectangle that is 12 inches long and 5 inches wide.



- A. $\frac{\sqrt{119}}{2}$ B. $\sqrt{\frac{17}{2}}$ C. 13
 D. $\sqrt{61}$ E. none of the above

25. The rectangular prism shown has a volume of 2400 cubic centimeters. It has a length of 20 cm and a width of 10 cm. Find the height of the prism, x .



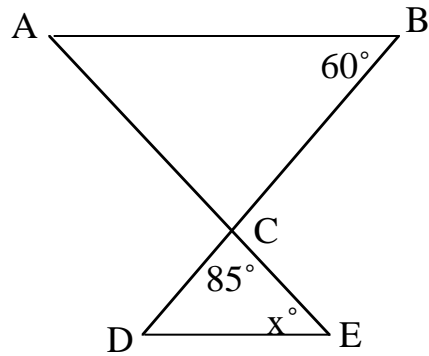
- A. 800 cm B. 10 cm C. 12 cm
- D. 2370 cm E. none of the above
26. What is the reference angle, in degrees, of $\frac{283\pi}{90}$?
- A. 26π B. 206 C. -26
- D. 386 E. none of the above
27. Find the value of the following if $x = 1$, $y = -1$ and $z = -1$.
- $$e^x e^y - \ln(e^x) + (e^x)^y - e^{\ln(x-z)} - \ln(e^{x+y}) + \ln\left(-\frac{x}{y}\right) - e^z$$
- A. -2 B. -3 C. e
- D. $\ln e$ E. none of the above
28. Give the output of the given program.
- ```

10 LET A=2.1×105
20 LET B=0.02A
30 LET A=B
40 IF A>100, GOTO 20
50 LET C=A+21
60 PRINT C
70 END

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- A. 84                      B. 105                      C. 21
- D. 21.02                      E. none of the above

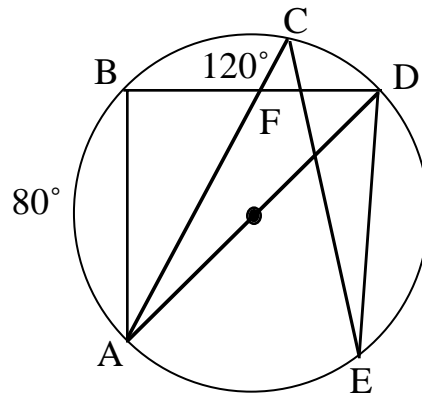


32. In the adjacent figure  $\overline{AB}$  is parallel to  $\overline{DE}$ . The measure of angle B is  $60^\circ$  and the measure of angle DCE is  $85^\circ$ . Find the measure of angle E.



- A.  $85^\circ$                       B.  $60^\circ$                       C.  $35^\circ$   
D.  $30^\circ$                       E. none of the above
33. Three playing cards are placed in a row as follows: The spade is to the right of the heart and the diamond. The 2 is to the left of the heart. The 9 is to the right of the 5. What is the middle card?
- A. 5 of spades                      B. 5 of hearts                      C. 2 of hearts  
D. 5 of diamonds                      E. none of the above
34. Convert  $342.17^\circ$  to DMS (Degree, Minute, Second) measure.
- A.  $342^\circ 10' 12''$                       B.  $342^\circ 1' 7''$                       C.  $342^\circ 10' 2''$   
D.  $342^\circ 12' 10''$                       E. none of the above
35. What is the decimal equivalent of the Roman numeral XXXIX?
- A. 41                      B. 38                      C. 39  
D. 40                      E. none of the above

36. The circle shown has a diameter  $\overline{AD}$ . The measure of minor arc  $AB$  is  $80^\circ$  and the measure of angle  $BFC$  is  $120^\circ$ . Find the measure of angle  $DEC$ .



- A.  $30^\circ$                       B.  $20^\circ$                       C.  $40^\circ$   
 D.  $10^\circ$                       E. none of the above
37. Find the exact value of the following:  
 $\sin 20^\circ \cos 10^\circ + \cos 20^\circ \sin 10^\circ + \cos 70^\circ \cos 20^\circ - \sin 70^\circ \sin 20^\circ$
- A.  $\frac{3}{2}$                       B.  $\frac{\sqrt{3}}{2}$                       C.  $\frac{1}{2}$   
 D.  $-\frac{1}{2}$                       E. none of the above
38. Find  $a + b + c$  where  $(a, b, c)$  is the solution to the following system of equations.

$$3a - 4b + 2c = 15$$

$$2a - b + c = 13$$

$$a + 2b - c = 5$$

- A. 2                      B. 22                      C. -2  
 D. 15                      E. none of the above

39. If  $b > 1$ ,  $\sin x > 0$ ,  $\cos x > 0$  and  $\log_b \sin x = a$ , which of the following expressions is equivalent to  $\log_b \cos x$ ?

A.  $2 \log_b(1 - b^{a/2})$     B.  $\sqrt{1 - a^2}$     C.  $b^{a^2}$

D.  $\frac{1}{2} \log_b(1 - b^{2a})$     E. none of the above

40. Evaluate the following:  $8^0 + 8^{1/3} + 8^{2/3} + 8^1 + 8^{4/3} + 8^{5/3} + \dots + 8^3$

A. 120    B. 1023    C. 1024

D. 90    E. none of the above

## 2002 Varsity Written Exam Answers

1. B
2. A
3. C
4. B
5. D
6. D
7. E (answer: 22)
8. D
9. D
10. A
11. A
12. D
13. B
14. B
15. D
16. C
17. C
18. B
19. A
20. C
21. B
22. B
23. B
24. E (answer:  $6\frac{1}{2}$  inches)
25. C
26. E (answer:  $26^\circ$ )
27. A
28. B
29. C
30. A
31. C
32. C
33. B
34. A
35. C
36. B
37. C
38. E (answer: 14)
39. D

40. B