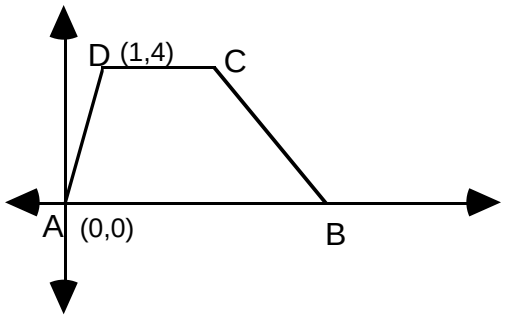
GEORGIA SOUTHERN UNIVERSITY
1997 MATHEMATICS TOURNAMENT
JUNIOR VARSITY WRITTEN EXAM

1. You own a stock that is traded on the New York Stock Exchange. On Monday it closed at \$23 per share, it fell \$3 on Tuesday and another \$6 on Wednesday, it rose \$2 on Thursday, and finished strongly on Friday by rising \$7. Determine the closing price on Friday.
   (a) 41
   (b) 23
   (c) 5
   (d) 0
   (e) none of the above

2. A lump of metal is submerged in a rectangular water tank 20 cm by 15 cm, raising the level of the water 0.35 cm. What is the volume of the metal?
   (a) 7 cm^3
   (b) 10.5 cm^3
   (c) 105 cm^3
   (d) 300 cm^3
   (e) $10,500 \text{ cm}^3$

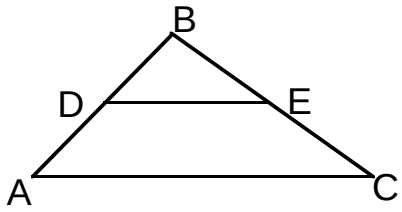
3. Solve for x: $5 + 3[1 + 2(2x - 3)] = 6(x + 5)$
   (a) $\frac{17}{7}$
   (b) -14
   (c) $\frac{30}{7}$
   (d) $\frac{20}{3}$
   (e) $\frac{11}{6}$

4. What is the slope of a line perpendicular to $\overline{AD}$?



(a) 3
(b) –4
(c) 4
(d) $\frac{1}{4}$
(e) $\frac{-1}{4}$

5. Points D and E are midpoints. If DE = 15 and AC = 3x + 15, then x = _____.



(a) 5
(b) 10
(c) 15
(d) 30
(e) none of the above

6. Given the equation of a circle, $x^2 + y^2 + 6x - 14y + 42 = 0$, find the center and radius of the circle.
(a) center (3,–7), radius = 4
(b) center (–3,7), radius = 4
(c) center (–3,7), radius = 16
(d) center (3,–7), radius = 16
(e) center (–3,–7), radius = 4

7. The sum of 5 consecutive even integers is 50. Find the product of the first and fifth integer.

(a) 72
(b) 76
(c) 84
(d) 96
(e) 120

8. If $f(x) = \sqrt{x+5}$, and $f(a) = 3$, find a.

(a) $2\sqrt{2}$
(b) $\left(3-\sqrt{5}\right)^2$
(c) 4
(d) $3-\sqrt{5}$
(e) –2

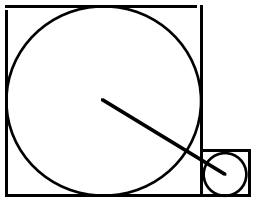
9. An electrician requires 12 hours to wire a house. The electrician's apprentice can wire a house in 16 hours. After working alone on one job for 4 hours, the electrician quits and the apprentice completes the task. How long does it take the apprentice to finish wiring the house?

(a) $10\frac{2}{3}$ hours
(b) 12 hours
(c) $14\frac{2}{3}$ hours
(d) $21\frac{1}{3}$ hours
(e) none of the above

10. The lengths of the dimensions of a rectangular solid are given by three consecutive even integers. Write a formula that will give the surface area of the solid.

(a) $2(x+2)^2$
(b) $4x^2+24x+32$
(c) $3x^2+12x+8$
(d) $x(x+2)(x+4)$
(e) $2\left(3x^2+12x+8\right)$

11. The areas of these two adjacent squares are 4 cm^2 and 196 cm^2. Find the length of the segment joining the centers of their inscribed circles.



(a) 10 cm
(b) 10.3 cm
(c) 10.6 cm
(d) 11 cm
(e) 11.4 cm

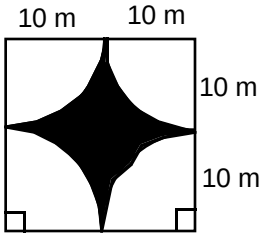
12. Factor: $2x^{2n} + 3x^n - 2$

(a) $\left(2x^n + 1\right)\left(x^n - 2\right)$
(b) $n(2x - 1)(x + 2)$
(c) $\left(2x^n - 1\right)\left(x^n + 2\right)$
(d) $5x^{3n} - 2$
(e) prime (can't be factored)

13. A wildlife mangement team estimates the number of bears in a forest by the capture-mark-recapture technique. Using live traps 30 bears are captured, marked and released. After a period of mixing, another 30 bears are captured and 5 marked bears from the first sample are found. Assuming the ratio of the total bear population to bears marked in the first sample is the same as the ratio of bears in the second sample to those marked in the second sample, estimate the bear population.

(a) 60
(b) 65
(c) 150
(d) 180
(e) 900

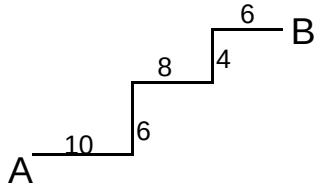
14. Find the area of the shaded region.



(a) $400 - 100\pi$ m^2
(b) $400 - 200\pi$ m^2
(c) $400 - 20\pi$ m^2
(d) $100 - 100\pi$ m^2
(e) $100 - 20\pi$ m^2

15. Find the point on the line $y = -2x + 4$ that is closest to the origin.
(a) $(0,4)$
(b) $(2,0)$
(c) $(1,2)$
(d) $\left(\frac{8}{3}, \frac{-4}{3}\right)$
(e) $\left(\frac{8}{5}, \frac{4}{5}\right)$

16. Determine the length of $\overline{AB}$.



(a) 34
(b) 28
(c) 26
(d) 24
(e) none of the above

17. Sam invested part of his $10,000 bonus in a fund that paid an 11% profit and invested the rest in stock that suffered a 4% loss. Find the amount of each investment if his overall net profit was $650.

(a) $7000 @11% and $3000 @ 4%

(b) $3000 @11% and $7000 @ 4%

(c) $6000 @11% and $4000 @ 4%

(d) $4000 @11% and $6000 @ 4%

(e) none of the above

18. A merchant is giving canned food to three different food banks. He gives the first food bank 1/3 of his food; he gives the second 1/4 of the remainder of his food; and he gives the third 1/5 of the food remaining after his first two stops. The total amount of food the merchant gives away is 72 cans. How many cans of food did he begin with?

(a) 288

(b) 180

(c) 120

(d) 93

(e) 72

19. Estimate to the nearest whole number the percent increase in the area of a circular pizza if the radius is increased from nine inches to ten inches.

(a) 10%

(b) 11%

(c) 19%

(d) 23%

(e) 25%

20. If each of these three operation signs, $+$, $-$, and $\times$, is used exactly once in the blanks in the expression
5 __ 4 __ 6 __ 3,
then the value of the result could equal which of the following?

(a) 9

(b) 10

(c) 15

(d) 16

(e) 19

21. A coin is tossed five times. What is the probability that the outcome will not be five tails?

(a) $\frac{1}{32}$

(b) $\frac{1}{2}$

(c) $\frac{31}{32}$

(d) $\frac{5}{2}$

(e) none of the above

22. One hundred umbrellas were sold on a rainy day. Five times as many red as blue were sold, three times as many green as blue, eighteen more black than blue, and fourteen fewer purple than black. Ten umbrellas were purple and twelve were white. How many red umbrellas were sold?

(a) 6

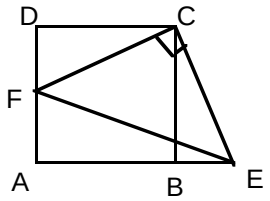
(b) 30

(c) 60

(d) 70

(e) none of the above

23. The area of square ABCD is 256. The area of right triangle CEF is 200. What is the length of $\overline{BE}$?



(a) 7.5

(b) 12

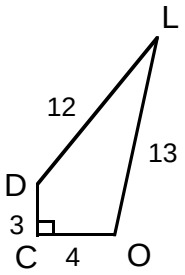
(c) 13

(d) 25.6

(e) 144

24. Which of the following numbers has the largest prime factor?
    (a) 39
    (b) 51
    (c) 77
    (d) 91
    (e) 121

25. What is the area of quadrilateral COLD?



    (a) 84
    (b) 72
    (c) 40
    (d) 38.6
    (e) 36

26. Tom and Jake went fishing. At day's end, Tom said to Jake, "If you give me one of your fish, then I'll have twice as many as you." Jake answered, "If you give me one of your fish, then we'll have the same number of fish." How many fish did Tom catch?
    (a) 4
    (b) 5
    (c) 6
    (d) 7
    (e) 8

27. Lois is two meters tall. A streetlight, ten meters away, casts a four-meter shadow of Lois. How tall is the streetlight?
    (a) 5 meters
    (b) 7 meters
    (c) 14 meters
    (d) 28 meters
    (e) none of the above

28. If Δ represents an operation defined as $a \Delta b = b^a + a^b$, find $(2 \Delta 1) \Delta 2$.

(a) 4
(b) 5
(c) 12
(d) 16
(e) 17

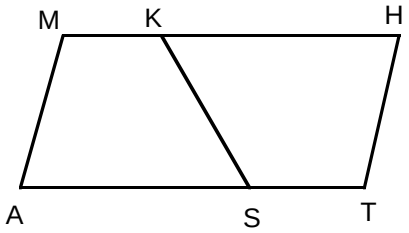
29. $8c^3 - (b + c)^3$ can be factored as the product of a binomial and a trinomial. The trinomial factor can be written in the form $Ab^2 + Ebc + Fc^2$, where A, E, and F are natural numbers. What is the value of $A + E + F$?

(a) 12
(b) 10
(c) 9
(d) 8
(e) 4

30. Find k if $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}} = \left(\frac{a}{b}\right)^k$.

(a) $\frac{1}{8}$
(b) $\frac{1}{2}$
(c) $\frac{3}{8}$
(d) $\frac{5}{8}$
(e) $\frac{-1}{2}$

31. MATH is a parallelogram in which $SK = 12$, $MK = \left(\frac{1}{3}\right)MH$, and $ST = \left(\frac{1}{3}\right)AT$. If the perimeter of MASK is 40, find the perimeter of MATH.

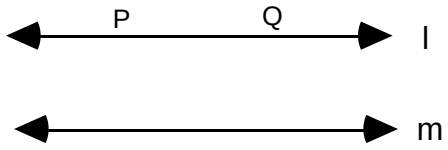


(a) 104
(b) 80
(c) 56
(d) 28
(e) none of the above

32. Find x if $\log_x 81 = 4$.

(a) 2
(b) 3
(c) 4
(d) 9
(e) none of the above

33. Lines l and m are parallel as shown. Points P and Q are on line l. The distance between lines l and m is less than the length of $\overline{PQ}$. How many different points, R, can be found on line m so that ΔPQR is isosceles?



(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

34. Find the sum of the real roots of $(x+1)(x+2)(x+4)(x-1)=72$.

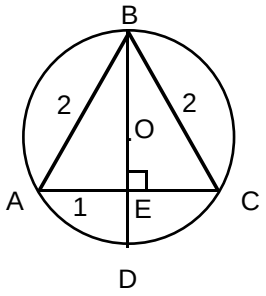
(a) 80
(b) 72
(c) 0
(d) –3
(e) –6

35. We interviewed forty-eight students about recycling paper (P), bottles (B), and cans (C). The following chart shows the number of students who do not recylcle one or a combination of these items. How many students recycle all three items?

P	B	C	PB	PC	BC	PBC
13	6	9	3	7	4	2

(a) 46
(b) 34
(c) 32
(d) 20
(e) 4

36. Equilateral triangle ABC is inscribed in the circle O, with $\overline{BE} \perp \overline{AC}$. Find BD.



(a) $\frac{\sqrt{3}}{3}$
(b) $\frac{2\sqrt{3}}{3}$
(c) $\sqrt{3}$
(d) $\frac{4\sqrt{3}}{3}$
(e) $\frac{5\sqrt{3}}{3}$

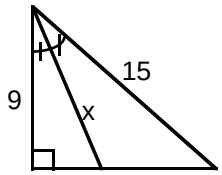
37. What is the value of x if $4^{20} + 4^{20} = 2^x$?

(a) 23
(b) 24
(c) 40
(d) 41
(e) 80

38. Let x and y be positive integers and $n = x^y$. If $n + \sqrt{n} + \sqrt[3]{n} = 76$, then x can not be equal to which of the following.

(a) 64
(b) 16
(c) 8
(d) 4
(e) 2

39. Find x.



(a) $\frac{3\sqrt{61}}{2}$
(b) $\frac{3\sqrt{109}}{2}$
(c) $\frac{15\sqrt{5}}{2}$
(d) $\frac{9}{2}$
(e) $\frac{9\sqrt{5}}{2}$

40. If $m + n = 3$ and $m^2 + n^2 = 6$, find the numerical value for $m^3 + n^3$.

(a) $\frac{27}{2}$
(b) 27
(c) 18
(d) 9
(e) $\frac{9}{2}$

GSU MATH TOURNAMENT
1997 JUNIOR VARSITY EXAM SOLUTIONS

1. B
2. C
3. D

4. E
5. A
6. B

7. C
8. C
9. A
10. E

11. A
12. C
13. D

14. A
15. E
16. C

17. A
18. C
19. D
20. E

21. C
22. B
23. B

24. B
25. E
26. D
27. B

28. E
29. A
30. C

31. C
32. B
33. E

34. D
35. C
36. D

37. D
38. B
39. E

40. A