

**GEORGIA SOUTHERN UNIVERSITY
HIGH SCHOOL MATHEMATICS TOURNAMENT
1997 VARSITY WRITTEN EXAM**

Name _____

School _____

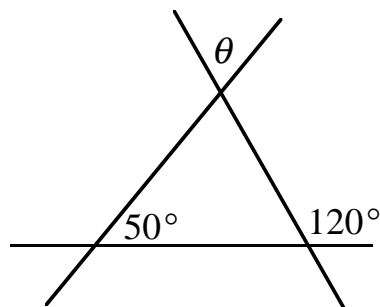
Directions:

1. Do not open this test booklet until you are told to do so.
2. Use only a #2 lead pencil.
3. No calculators, slide rules, notes or other aids of any kind may be used.
4. Scratch paper is stapled to the back of the test booklet.
5. This is a 40 question multiple-choice exam. You will be allotted 90 minutes to complete the exam.
6. Geometric figures are not necessarily drawn to scale.
7. Your score will be determined by the formula $40 + 4R - W$ where R = number of questions answered correctly and W = number of questions answered wrong. There is no penalty for questions left unanswered.
8. Tie-breakers will be taken from the written exam in order of difficulty. This order will be determined by the number of people that answered each question correctly, with the question(s) correctly answered by the fewest people considered first.

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1. Find the measure of $\angle \theta$ in the figure.

- A. 50°
- B. 60°
- C. 70°
- D. 110°
- E. None of the above



2. What is the value of $(\sqrt{5} - 4i)$ times its conjugate?

- A. $-11 - 8i\sqrt{5}$
- B. 1
- C. 9
- D. 21
- E. None of the above

3. What is the $\cot 270^\circ$?

- A. -1
- B. undefined
- C. 1
- D. $-\frac{\sqrt{3}}{2}$
- E. None of the above

4. The arithmetic mean of x and $4x$ is 20. Compute x .

- A. $\pm\sqrt{5}$
- B. 4
- C. 8
- D. $\frac{40}{3}$
- E. None of the above

5. A group of cars, including Joe's car, is racing around a circular race track. If one-third of the number of cars located in front of Joe added to three quarters of the number of cars located behind Joe equals the total number of cars in the race, how many cars were racing?

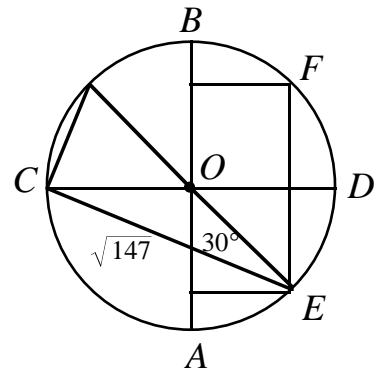
- A. 12
- B. 13
- C. 24
- D. 25
- E. None of the above

6. A man 6 feet tall has been walking away from a streetlight. When he is 10 feet from the light, he stops. If his shadow is 11 feet long, how high is the streetlight in feet?

- A. $\frac{60}{11}$
- B. $\frac{126}{11}$
- C. $\sqrt{187}$
- D. 11
- E. None of the above

7. Find $|OF|$ if $|CD|=14$ and O is the center of the circle.

- A. 14
- B. $7\sqrt{3}$
- C. $\frac{14\sqrt{3}}{3}$
- D. $7\sqrt{2}$
- E. None of the above



8. Find x given $222_{\text{base } x} = 114_{\text{ten}}$ if $x > 0$.

- A. 7
- B. 8
- C. 55
- D. 56
- E. None of the above

9. Which of the following functions is even?

- A. $f(x) = x^2 \sin x$
- B. $g(x) = \tan x + 3$
- C. $m(x) = -\csc x$
- D. $k(x) = 2 \cos x$
- E. None of the above

10. Assume that, for a certain school, it is true that

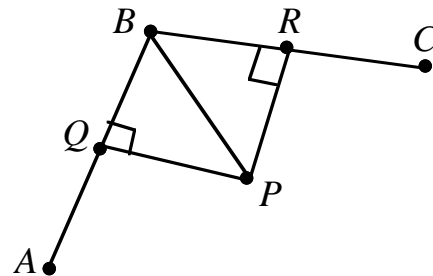
I: Some students are not honest.

II: All fraternity members are honest.

Which of the following statements is a necessary conclusion?

- A. Some students are fraternity members.
- B. Some fraternity members are not students.
- C. Some students are not fraternity members.
- D. No fraternity member is a student.
- E. None of the above

11. Q is the midpoint of segment AB . R is the midpoint of segment BC . Given that $|AB|=18$, $|PQ|=12$ and $|PR| = 2\sqrt{26}$, find $|BC|$.



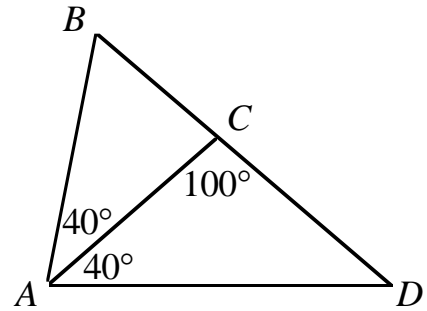
- A. 11
- B. 18
- C. 22
- D. $4\sqrt{26}$
- E. None of the above

12. Evaluate $\begin{vmatrix} 15 & -5 \\ 8 & -1 \end{vmatrix}$.

- A. -55
- B. 17
- C. 25
- D. 29
- E. None of the above

13. Find $|AD|$ given that $|AC| \approx 7.91$, $|AB| = 9$ and $|BC| \approx 5.87$.

- A. $\frac{\sqrt{3} \sin 40^\circ}{18}$
- B. $\frac{9\sqrt{3}}{2 \sin 40^\circ}$
- C. $\frac{9 \sin 40^\circ}{2\sqrt{3}}$
- D. $\frac{2 \sin 40^\circ}{9\sqrt{3}}$
- E. None of the above

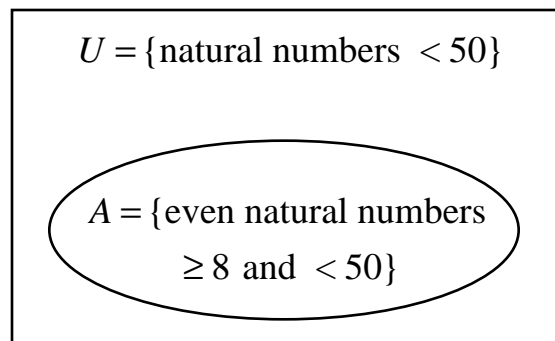


14. If $f(x) = 3$ and $g(x) = 24x^2 - 7$, find $f(g(-9))$.

- A. 3
- B. 209
- C. $-27(24x^2 - 7)$
- D. $3(24x^2 - 16)$
- E. None of the above

15. Find the cardinality of the complement of A.

- A. 21
- B. 27
- C. 28
- D. 29
- E. None of the above



16. Find the area above the x -axis bounded by the two lines $y = 3x$ and $y = 6x - 12$.

- A. 4
- B. 12
- C. 16
- D. 24
- E. None of the above

17. Find a_{27} if $a_n = \begin{cases} n^3 + n & n < 0 \\ (n+2)(n-2) & 0 \leq n < 16. \\ \sqrt[3]{n} & n \geq 16 \end{cases}$

- A. 3
- B. 27
- C. 725
- D. 19,710
- E. None of the above

18. A social club contains 7 women and 4 men. The club wants to select a committee of 3 members to represent it at a state convention. How many of the possible committees that could be chosen contain at least one man?

- A. 4
- B. 84
- C. 130
- D. 14,112
- E. None of the above

19. What will be the output of this program?

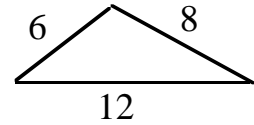
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10 LET A=1
20 LET B=2
30 LET C=A*B
40 IF C>200, GOTO 80
50 LET A=B
60 LET B=C
70 GOTO 30
80 D=C÷64
90 PRINT D
100 END

```

- A. $\frac{1}{32}$
- B. 2
- C. 4
- D. 256
- E. None of the above

20. Find the area of this triangle.



- A. $\sqrt{455}$
- B. $\sqrt{35}$
- C. $12\sqrt{7}$
- D. 42
- E. None of the above

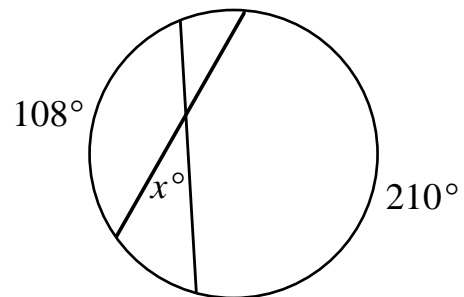
21. For all positive numbers x , y and z , simplify the product

$$(x + y + z)^{-1}(x^{-1} + y^{-1} + z^{-1})(xy + yz + zx)^{-1}[(xy)^{-1} + (yz)^{-1} + (zx)^{-1}].$$

- A. $x^{-2}y^{-2}z^{-2}$
- B. $x^{-2} + y^{-2} + z^{-2}$
- C. $\frac{1}{xy + yz + zx}$
- D. $\frac{1}{xyz}$
- E. None of the above

22. Find x .

- A. 21
- B. 28
- C. 42
- D. 72
- E. None of the above



23. Find x if $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 16 \end{bmatrix}$.

- A. $\frac{5}{2}$
- B. $\frac{11}{2}$
- C. 6
- D. 7
- E. None of the above

24. If (a, b) and (c, d) are two points on the line with equation $y = mx + k$, find the distance between (a, b) and (c, d) , in terms of a , c and m .

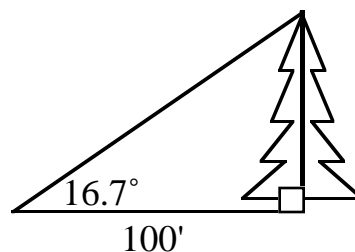
- A. $|a - c|\sqrt{1 + m^2}$
- B. $|a + c|\sqrt{1 + m^2}$
- C. $|a - c||m|$
- D. $|a - c|(1 + m^2)$
- E. None of the above

25. If $a = \frac{1}{2}$ and $(a + 1)(b + 1) = 2$, find the radian measure of $\arctan a + \arctan b$.

- A. $\frac{\pi}{3}$
- B. $\frac{\pi}{4}$
- C. $\frac{\pi}{5}$
- D. $\frac{\pi}{6}$
- E. None of the above

26. Find the tree's height to the nearest foot given $\sin 16.7^\circ \approx 0.29$ and $\cos 16.7^\circ \approx 0.96$.

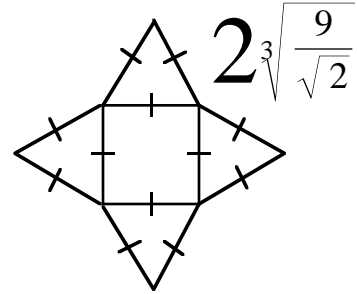
- A. 29
- B. 30
- C. 95
- D. 96
- E. None of the above



27. Line L passes through points $(-1, 3)$ and $(-3, a)$. Its slope is -13 . Find the value of a .

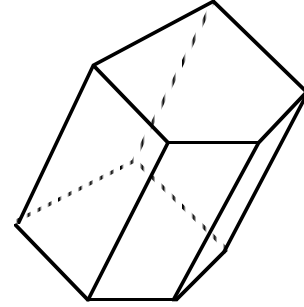
- A. $\frac{41}{13}$
- B. 23
- C. 26
- D. 29
- E. None of the above

28. Find the volume of a pyramid formed from this model.



- A. 12
B. $12\sqrt[3]{\frac{3}{2}}$
C. $24\sqrt[3]{\frac{9}{\sqrt{2}}}$
D. 36
E. None of the above
29. Solve for x if $\cos^2 x - 1 = 0$ and $0 \leq x < 2\pi$.
- A. $\frac{\pi}{4}, \frac{3\pi}{4}$
B. $\frac{\pi}{2}, \frac{3\pi}{2}$
C. 1, -1
D. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
E. None of the above
30. The number of gnats $G(x)$, in millions, in a certain area of South Georgia depends, among other things, on the average July temperature, x , and may be approximated by the formula $G(x) = 364x - 2x^2$. Find the temperature that produces the maximum number of gnats.
- A. 88°
B. 91°
C. 93°
D. 97°
E. None of the above
31. Write $\sin x \cos 3x + \cos x \sin 3x$ in terms of a single trigonometric function.
- A. $\tan 2x$
B. $\cos 4x$
C. $\sin 4x$
D. $-\sin 2x$
E. None of the above

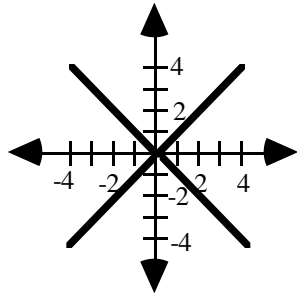
32. If the volume of this prism is 12 cubic units, what would the volume of a pyramid with the same base and height be?



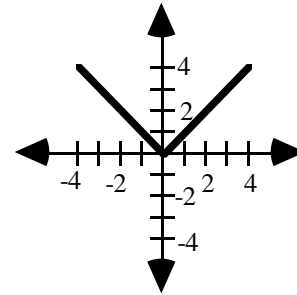
- A. 4
- B. 8
- C. 12
- D. 36
- E. None of the above

33. Which of the following is the graph of $f(x) = \sqrt{x^2}$?

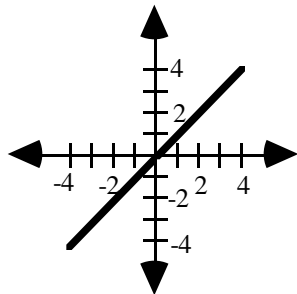
A.



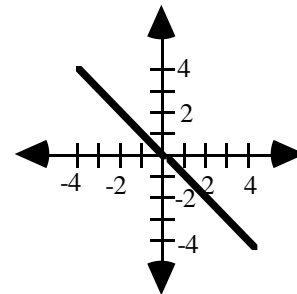
B.



C.



D.

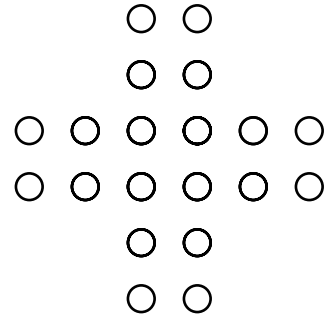


- E. None of the above

34. The supplement of an angle is 12 more than twice the angle's complement. What is the angle's measure?

- A. 4
- B. 12
- C. 102
- D. 174
- E. None of the above

35. If the rows and columns of pennies are perpendicular, and the pennies are all equally spaced, how many squares are formed by joining centers of the pennies?

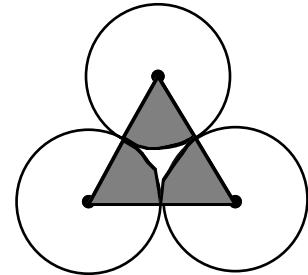


- A. 9
 B. 13
 C. 17
 D. 21
 E. None of the above
36. If $f(x) = ke^{ax} + b$, find $f^{-1}(x)$.

- A. $\frac{\ln(x - b)}{ka}$
 B. $\frac{x - b}{ke^a}$
 C. $\frac{1}{ke^{ax} + b}$
 D. $\frac{1}{a} \ln\left(\frac{x - b}{k}\right)$
 E. None of the above

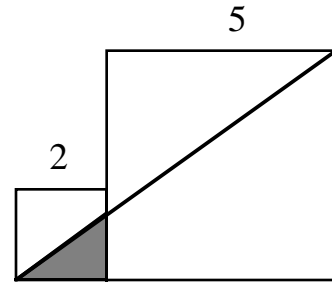
37. The radii of these tangent circles are each $\frac{4}{\sqrt{\pi}}$.

What is the area of the shaded region inside the triangle whose vertices lie on the circles' centers?



- A. $\frac{4\sqrt{\pi}}{3}$
 B. $\frac{4\sqrt{3}}{\pi}$
 C. 4
 D. 8
 E. None of the above

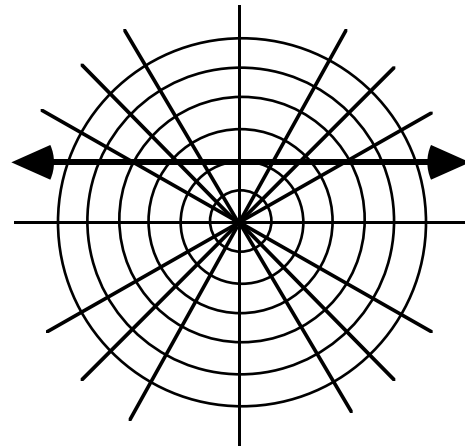
38. The two squares have dimensions as indicated. What is the area of the shaded triangle?



- A. $\frac{10}{7}$
 B. 2
 C. $\frac{\sqrt{74}}{2}$
 D. 19
 E. None of the above
39. The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. Find the cosine of the smallest angle.

- A. $\frac{3}{4}$
 B. $\frac{7}{10}$
 C. $\frac{2}{3}$
 D. $\frac{9}{14}$
 E. None of the above

40. What is the polar equation of the graph shown in the polar coordinate system?



- A. $r \cos \theta = 2$
 B. $r \sin \theta = 2$
 C. $r = 2 \sin \theta$
 D. $r = 2 \cos \theta$
 E. None of the above

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ANSWERS

- | | | | |
|-----|-----------------------|-----|-------------------------------|
| 1. | C | 21. | A |
| 2. | D | 22. | A |
| 3. | E (correct answer: 0) | 23. | C |
| 4. | C | 24. | A |
| 5. | B | 25. | B |
| 6. | B | 26. | B |
| 7. | E (correct answer: 7) | 27. | D |
| 8. | A | 28. | A |
| 9. | D | 29. | E (correct answer: 0, π) |
| 10. | C | 30. | B |
| 11. | C | 31. | C |
| 12. | C | 32. | A |
| 13. | B | 33. | B |
| 14. | A | 34. | B |
| 15. | C | 35. | D |
| 16. | B | 36. | D |
| 17. | A | 37. | D |
| 18. | C | 38. | A |
| 19. | C | 39. | A |
| 20. | A | 40. | B |