

**GEORGIA SOUTHERN UNIVERSITY
INVITATIONAL MATHEMATICS TOURNAMENT
2001 VARSITY WRITTEN EXAM**

Name _____

School _____

Directions:

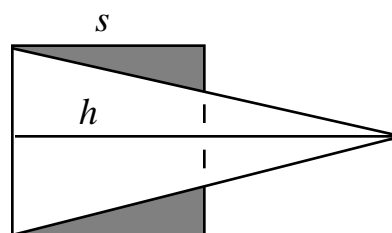
1. Do not open this test booklet until you are told to do so.
2. Use only a #2 lead pencil.
3. No calculators, slide rules, notes or other aids of any kind may be used.
4. Scratch paper is stapled to the back of the test booklet.
5. This is a 40 question multiple-choice exam. You will be allotted 90 minutes to complete the exam.
6. Geometric figures are not necessarily drawn to scale.
7. Your score will be determined by the formula $40 + 4R - W$ where R = number of questions answered correctly and W = number of questions answered wrong. There is no penalty for questions left unanswered.
8. Tie-breakers will be taken from the written exam in order of difficulty. This order will be determined by the number of people that answered each question correctly, with the question(s) correctly answered by the fewest people considered first.

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1. Which of these rational expressions equals -1 ?

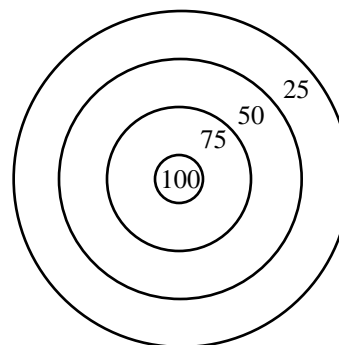
- A. $\frac{x-3}{x+3}$ B. $\frac{x+3}{-x+3}$ C. $\frac{x-3}{3-x}$
- D. $\frac{x-3}{-x-3}$ E. none of the above

2. The isosceles triangle and the square have identical areas. Express the altitude h as a function of the base s .



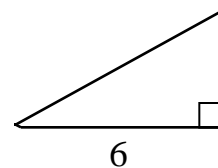
- A. $\frac{1}{2}s$ B. s C. $2s$
- D. $4s$ E. none of the above

3. The dart board has a 7" radius with each region of the dart board 2" wide, except the bull's eye which has a 1" radius. If a dart is thrown at random and is guaranteed to hit the board, what is the probability that it will hit the ring worth 50 points?



- A. $\frac{15}{49}$ B. $\frac{16}{49}$ C. $\frac{2}{7}$
- D. $\frac{1}{7}$ E. none of the above

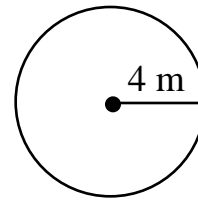
4. Find $(-\sqrt{3} + i)^6$.
- A. $64 + 64\sqrt{3}i$ B. $-64 + 64\sqrt{3}i$
- C. $-64 - 64\sqrt{3}i$ D. -64 E. $-64 + i$
5. The office team is out racing at the skating rink during a long lunch hour. Samantha beat Jim. Louise was not last. Dennis was beaten by Jack and Louise, in that order. Jack lost to Jim. Who won?
- A. Jim B. Louise C. Jack
- D. Dennis E. none of the above
6. Find the length of the arc of a circle with radius 25 cm and central angle measure 42° .
- A. $\frac{35}{6}$ B. 21 C. $\frac{35}{3}$
- D. 49 E. none of the above
7. Find a real number x such that $x = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$.
- A. 3 B. $2 + 3\sqrt{2}$ C. 4 or 1
- D. 4 E. none of the above
8. Find the area of this right triangle in square units if its perimeter is 18 units.



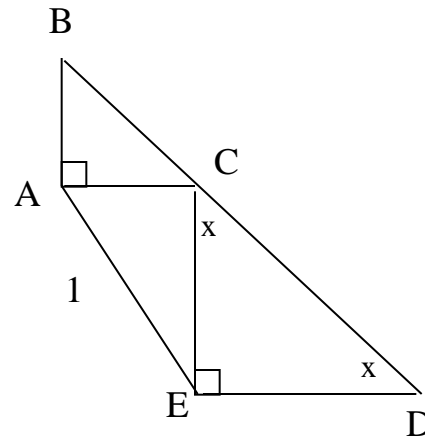
- A. $6\sqrt{3}$ B. 12 C. 18
- D. 13.5 E. none of the above

9. Determine k so that $4x^2 - kx + 1 = 0$ has one real solution.
- A. $k = 0$ B. $k = 4$ C. $k = -4$
- D. $k = \frac{1}{4}$ E. $k = 4$ and $k = -4$
10. Two identical oil pipelines are circular cylinders of radius 6. The Environmental Protection Agency has ordered that they be replaced with a single pipeline with the same capacity. If the new pipeline is also to be a circular cylinder, what must its radius be?
- A. $6\sqrt{2}$ B. $6\sqrt{3}$ C. 9
- D. 12 E. none of the above
11. If $a = \log_8 225$ and $b = \log_2 15$, which of these statements is true?
- A. $a = \frac{b}{2}$ B. $a = \frac{2b}{3}$ C. $a = b$
- D. $b = \frac{a}{2}$ E. $a = \frac{3b}{2}$
12. If $S = 1! + 2! + 3! + \dots + 99!$, find the units' digit in the value of S .
- A. 9 B. 8 C. 5
- D. 3 E. 0
13. Two fair dice are tossed and they do not show "doubles." What is the probability that one die shows a six?
- A. $\frac{1}{6}$ B. $\frac{5}{18}$ C. $\frac{1}{3}$
- D. $\frac{5}{6}$ E. none of the above

14. What is the distance traveled in meters after $2\frac{1}{2}$ revolutions?



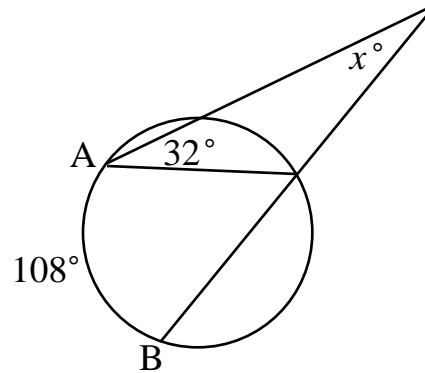
- A. 20 B. 10 C. 30
 D. 40 E. none of the above
15. Given that points B, C, and D are collinear, $AE = 1$ and $m \angle ECD = m \angle EDC$ find the sum of the areas of the right triangles ABC and ECD.



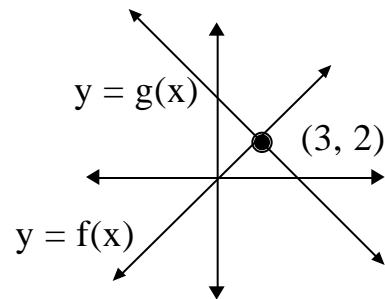
- A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 2
 D. 1 E. none of the above
16. If θ is an acute angle and $\sin \frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, find $\tan \theta$.

- A. x B. $\frac{1}{x}$ C. $\frac{\sqrt{x-1}}{x+1}$
 D. $\frac{\sqrt{x^2-1}}{x}$ E. $\sqrt{x^2-1}$

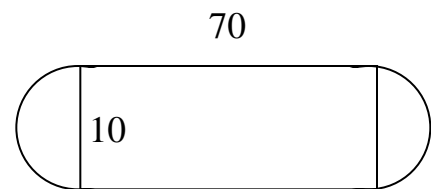
24. In the adjacent figure arc AB has a measure of 108° , find x .



- A. 22 B. 76
D. 40 E. none of the above
25. Given below is the graph of two linear functions $y = f(x)$ and $y = g(x)$. Use the graph to solve $f(x) < g(x)$. Express your answer in interval notation.

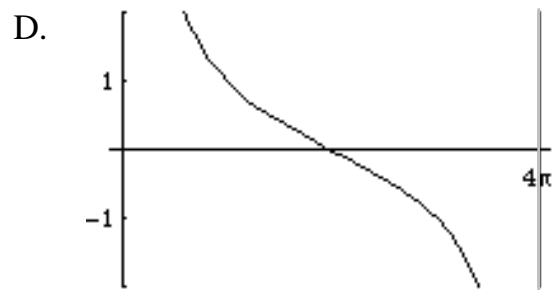
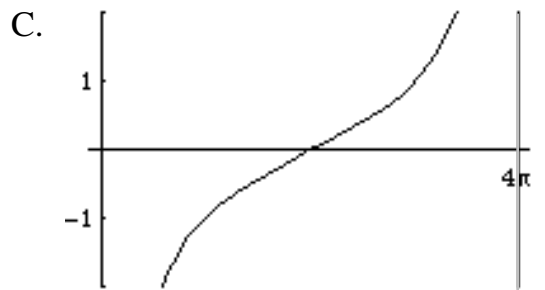
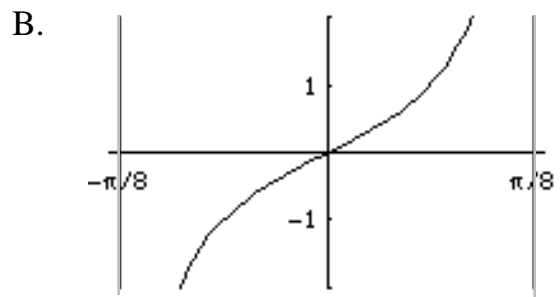
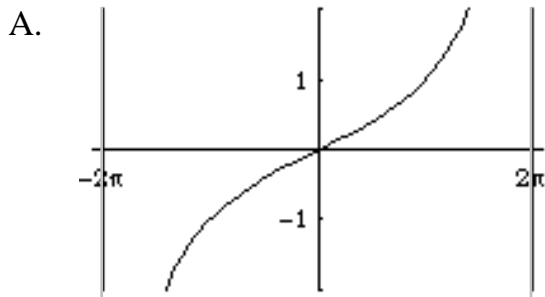


- A. $[2, \infty)$ B. $(-\infty, 3]$ C. $(3, \infty)$
D. $[3, \infty)$ E. none of the above
26. A race track is constructed by placing a semi-circle at each end of a rectangle that is 70 yards long. The diameter of the circle from which the semi-circle is formed is 10 yards. What is the area in square yards of the inside of the track?



- A. $10 + 700$ B. $100 + 700$ C. $25 + 700$
D. 700 E. none of the above

31. Which of the following shows one period of the graph of $y = \tan \frac{x}{4}$?



E. None of the above

32. Harper and Rose Lee can get three tiny bags of jellybeans and two tiny bags of chocolates for 24¢ , which was under their limit of a quarter. They could also get four tiny bags of chocolates and two tiny bags of jellybeans for the same 24¢ . How much did each tiny bag of chocolates cost?

A. 2¢

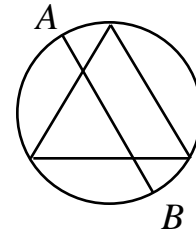
B. 6¢

C. 3¢

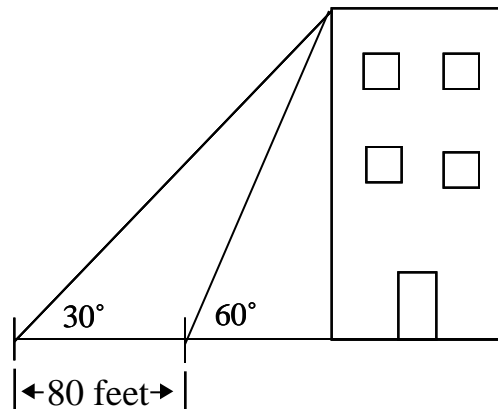
D. 4¢

E. none of the above

33. Diameter AB is 7 inches. An equilateral triangle is inscribed. Determine the triangle's area in square inches.

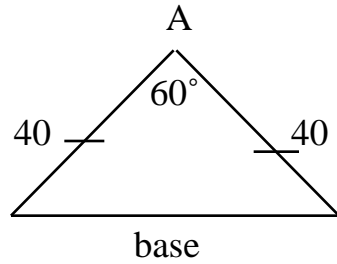


- A. $\frac{49\sqrt{3}}{16}$ B. $\frac{147\sqrt{3}}{16}$ C. $\frac{21\sqrt{3}}{16}$
- D. $\frac{147}{16}$ E. none of the above
34. Which property is illustrated in the following statement:
 $(-5 + 7)(3 + 4) = (-5 + 7) \cdot 3 + (-5 + 7) \cdot 4$
- A. distributive property B. associative property
- C. commutative property D. inverse property
- E. none of the above
35. Find the height in feet of a building if the angle to the top of the building from the ground changes from 30° to 60° as an observer moves 80 feet toward the building.



- A. $20\sqrt{3}$ B. 40 C. $40\sqrt{3}$
- D. 20 E. none of the above

36. The adjacent figure shows an isosceles triangle with congruent sides of 40 cm with a vertex angle A of measure 60° . Find the length of the altitude from vertex A to the base opposite A.



- A. 20 B. $20\sqrt{2}$ C. $\frac{80\sqrt{3}}{3}$
- D. $20\sqrt{3}$ E. none of the above
37. Given the set $X = \{-5, -0.245, 0, \frac{14}{7}, \sqrt{2}, \sqrt{81}\}$ which of the following subsets of X contain all the rational numbers in X.
- A. $-5, -0.245, 0, \frac{14}{7}$ B. $\{\sqrt{2}, \sqrt{81}\}$
- C. $-5, -0.245, 0, \frac{14}{7}, \sqrt{81}$ D. $\{\sqrt{2}\}$
- E. none of the above
38. For θ in the first quadrant, if $\sec \theta = \frac{5}{4}$ find $\sin \theta$.
- A. $\frac{1}{5}$ B. $\frac{4}{5}$ C. $\frac{3}{5}$
- D. $\frac{2}{5}$ E. none of the above

39. A rectangular solid with a square base has volume 40 cubic meters and surface area 72 square meters. If its length is a , height is b and width is c where a , b , and c are rational numbers then find $a + b + c$.

- A. $\frac{21}{2}$ B. 12 C. $\frac{118}{9}$
D. $24\sqrt[3]{5}$ E. none of the above

40. Evaluate the following.

$$100^2 - 99^2 + 98^2 - 97^2 + \cdots - 3^2 + 2^2 - 1^2$$

- A. 999 B. 5050 C. 5000
D. 50 E. none of the above

Answers 2001 Varsity Test

- | | | | | |
|-----|---|-----|---|-------------|
| 1. | C | 21. | E | (answer: 8) |
| 2. | C | 22. | C | |
| 3. | B | 23. | B | |
| 4. | D | 24. | A | |
| 5. | E | 25. | D | |
| 6. | A | 26. | C | |
| 7. | D | 27. | A | |
| 8. | D | 28. | A | |
| 9. | E | 29. | E | |
| 10. | A | 30. | C | |
| 11. | B | 31. | A | |
| 12. | D | 32. | C | |
| 13. | C | 33. | B | |
| 14. | A | 34. | A | |
| 15. | B | 35. | C | |
| 16. | E | 36. | D | |
| 17. | D | 37. | C | |
| 18. | D | 38. | C | |
| 19. | B | 39. | A | |
| 20. | B | 40. | B | |

