

**GEORGIA SOUTHERN UNIVERSITY
INVITATIONAL MATHEMATICS TOURNAMENT
2006 VARSITY WRITTEN EXAM**

Name _____

School _____

Directions:

1. Do not open this test booklet until you are told to do so.
2. Use only a #2 lead pencil.
3. No calculators, slide rules, notes or other aids of any kind may be used.
4. Scratch paper is stapled to the back of the test booklet.
5. This is a 40 question multiple-choice exam. You will be allotted 90 minutes to complete the exam.
6. Geometric figures are not necessarily drawn to scale.
7. Your score will be determined by the formula $40 + 4R - W$ where R = number of questions answered correctly and W = number of questions answered wrong. There is no penalty for questions left unanswered.
8. Tie-breakers will be taken from the written exam in order of difficulty. The order will be determined by the number of people that answered each question correctly, with the question(s) correctly answered by the fewest people considered first.

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1. What three things are accepted as true without verification or proof?
 - A. definitions, theorems and postulates
 - B. theorems, postulates and undefined terms.
 - C. conclusions, hypotheses, and postulates
 - D. definitions, postulates, undefined terms
 - E. None of the above

2. If TEN=20-5-14 and MEN=13-5-14, what do WOMEN equal by the same logic?
 - A. 22-15-13-5-14
 - B. 23-16-13-5-14
 - C. 27-19-20-5-14
 - D. 23-15-13-5-14
 - E. None of the above

3. Which of the following is true about the graph of the secant function on the interval $\left(\frac{3\pi}{2}, 2\pi\right)$?
 - A. It decreases.
 - B. It increases.
 - C. It is constant.
 - D. It is a straight line.
 - E. It is undefined.

4. Find the value of x so the line that passes through (x, 5) and (12, 2) is perpendicular to the line that passes through (0,1) and (2, 7).
 - A. 4
 - B. -3
 - C. 15
 - D. 5
 - E. None of the above

5. How many sides does a regular polygon have if each of its interior angles measure 162° ?
 - A. 10
 - B. 12
 - C. 17
 - D. 20
 - E. None of the above

6. If 55 percent of the lambs born are male and 90 percent of them survive the first year, what is the fewest number of lambs that must be born to have 100 male lambs living at the end of the first year?

A. 203 B. 202 C. 155
D. 50 E. None of the above

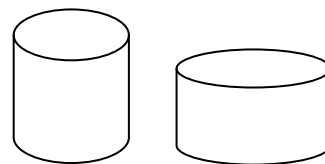
7. Which of the following is a point in the first quadrant where the maximum value of $y = 4 \sin\left(\frac{\pi x}{2}\right) + 3$ occurs.

A. (1, 7) B. (1, 3) C. (2, 3)
D. (0, 7) E. (2, 7)

8. If $\frac{x-7}{x-8} - \frac{x-8}{x-9} = \frac{x-4}{x-5} - \frac{x-5}{x-6}$ find $\frac{-x^2 + 2x}{2x+1}$.

A. $\frac{13}{63}$ B. $\frac{63}{13}$ C. $-\frac{7}{3}$
D. $-\frac{3}{7}$ E. None of the above

9. A 3×4 paper rectangle can be curled to form a cylinder in two ways: a tall cylinder and a short cylinder. Find the ratio of the volumes of the tall cylinder to the short cylinder.

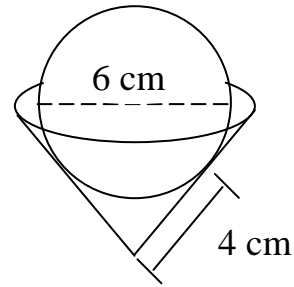


A. $\frac{3}{4}$ B. $\frac{9}{16}$ C. 1
D. $\frac{1}{2}$ E. None of the above

10. In the ancient Mayan culture, time was counted simultaneously by a 365-day solar year and a 260-day ritual calendar. Suppose that a year begins with day 1 on both calendars. How many solar years will pass until this situation occurs again?

A. 5 B. 73 C. 52
D. 105 E. None of the above

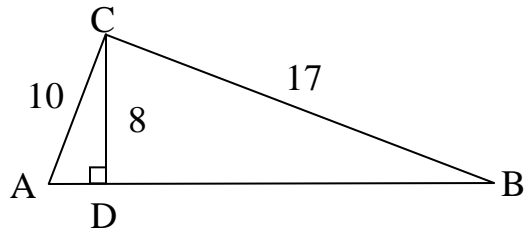
15. A sphere with a diameter 6 centimeters sits in a cone so that the circle of tangency is 4 centimeters up the cone's side from the vertex. How much liquid can be under the sphere if the sphere touches the liquid only at the very bottom of the sphere?



- A. $\frac{3\pi}{2} \text{ cm}^3$ B. $\frac{15\pi}{4} \text{ cm}^3$ C. $6\pi \text{ cm}^3$
 D. $\frac{3\pi}{4} \text{ cm}^3$ E. None of the above
16. How many distinct solutions exist for x in the equation

$$x^{(x+1)^2} = x^{16}?$$

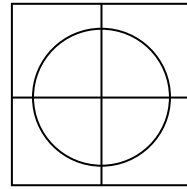
- A. 2 B. 5 C. 3
 D. 4 E. None of the above
17. Find the area of triangle ABC given $AC = 10$, $BC = 17$ and $DC = 8$, and \overline{DC} is perpendicular to \overline{AB} .



- A. 84 B. 60 C. 24
 D. 42 E. None of the above
18. If a clock strikes the proper number of chimes each hour on the hour, how many times will it chime in a month of 30 days?
- A. 9000 B. 8280 C. 3960
 D. 9360 E. None of the above

19. Find the length of the arc that subtends a central angle of 75° in a circle of radius 20 inches.
- A. $\frac{25}{3}$ inches B. $\frac{\pi}{24}$ inches C. 1500 inches
D. $\frac{25\pi}{3}$ inches E. None of the above
20. What is the axis of symmetry for $x = 2y^2 - 28y + 30$?
- A. $x = 15$ B. $y = 15$ C. $y = 7$
D. $x = 4y - 28$ E. None of the above
21. To which of the following does $\csc 2x + \cot 2x$ simplify?
- A. $\cot x$ B. $\tan^{-1} x$ C. $\tan x$
D. $\sec 2x$ E. None of the above
22. Find the sum of the following:
 $\log_2 8 + \log_4 8 + \log_8 8 + \log_{16} 8 + \log_{32} 8 + \log_{64} 8$.
- A. $\log_{126} 48$ B. 174 C. $\frac{126}{48}$
D. $\frac{147}{20}$ E. None of the above
23. Find the smallest number that when divided by 10 gives a remainder of 9, when divided by 9 gives a remainder of 8, when divided by 8 gives a remainder of 7, and so on, down to a remainder of 1 when the number is divided by 2.
- A. 2520 B. 4949 C. 2519
D. 5039 E. None of the above

24. What is the area of the circle that passes through the centers of the four unit squares, as shown?

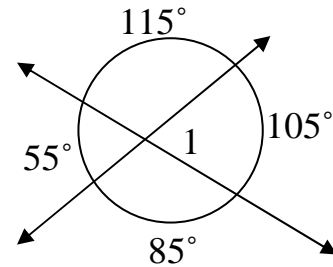


- A. π B. $\frac{1}{2}\pi$ C. $\frac{1}{4}\pi$
 D. $\frac{\sqrt{2}}{2}\pi$ E. None of the above
25. In a room of twenty students, every person shakes hands with exactly half the people in the room. What is the total number of handshakes?
- A. 200 B. 100 C. 150
 D. 50 E. None of the above
26. Evaluate $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$.
- A. $\frac{\sqrt{3}}{2}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{2}}{2}$
 D. $-\frac{\sqrt{2}}{2}$ E. None of the above
27. The supplement of an angle is 78° less than twice the supplement of the complement of the angle. Find the measure of the angle.
- A. 102° B. 26° C. 24°
 D. 146° E. None of the above
28. Find the value of $\sqrt{A+B} + \sqrt{C-D} - E$ where A is the number of zeros in one billion, B is the number of prime numbers less than 55, C is the number of the day of the year in 1941 for August 17, D is the greatest common divisor of 1492 and 1992, and E is the units digit of 92^{12} .
- A. 14 B. 13 C. 12
 D. 11 E. None of the above

29. In triangle XYZ, the sides have the following measures: $x = 3$ units, $y = 5$ units and $z = 7$ units. Find the measure of the biggest angle.

- A. 60° B. 30° C. 150°
 D. 120° E. None of the above

30. Shown is a circle and a pair of intersecting lines. The lines divide the circle into four arcs with the measures shown. Find the measure of angle 1.



- A. 80° B. 25° C. 105°
 D. 100° E. None of the above

31. If the retail price for a book is 25 percent more than its cost to the seller, what percent of the final price is profit?

- A. 25 B. 5 C. 15
 D. 20 E. None of the above

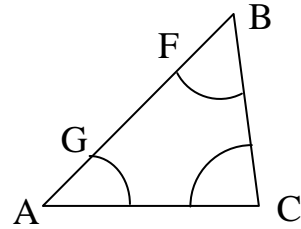
32. Simplify $[3(\cos 50^\circ + i \sin 50^\circ)]^2 \cdot [4(\cos 10^\circ + i \sin 10^\circ)]^2$.

- A. $48(\cos 60^\circ + i \sin 60^\circ)$ B. $144(\cos 60^\circ + i \sin 60^\circ)$
 C. $48(\cos 120^\circ + i \sin 120^\circ)$ D. $144(\cos 120^\circ + i \sin 120^\circ)$
 E. None of the above

33. Find y in the sequence 7, x , y , z , w , 11 if each term (except for x) is the sum of the two preceding terms.

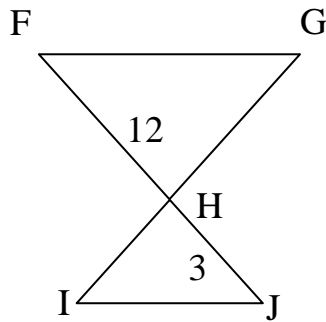
- A. 5 B. 1 C. 2
 D. -2 E. None of the above

34. In the acute triangle ABC , $AC = 16\sqrt{2}$, $FG = 6$, $m\angle BAC = 45^\circ$, and the sector at each vertex of the triangle has radius of $r = 8$. Find the area of the region inside the triangle but outside the sectors.



- A. $176 - 64\pi$ B. $176 - 32\pi$ C. $352 - 64\pi$
 D. $48 - 32\pi$ E. None of the above

35. In the figure shown \overline{FG} is parallel to \overline{IJ} . If $FH = 12$, $HJ = 3$ and the perimeter of $\triangle HIJ$ is 13 units then what is the perimeter of $\triangle FGH$?



- A. 36 B. 25 C. 52
 D. 48 E. None of the above
36. From a point on the ground 20 feet away from a vertical building the angle of elevation to the top of the building is measured as 60° . How tall is the building?
- A. $\frac{2\sqrt{3}}{3}$ feet B. $20\sqrt{3}$ feet C. 10 feet
 D. $\frac{20\sqrt{3}}{3}$ feet E. None of the above
37. Sally had a coin purse with fifty coins, totaling exactly \$1.00. Unfortunately, while counting them, she dropped one coin behind the radiator. If each possible collection of coins has equal probability of occurring what is the probability that the coin dropped was a penny?
- A. 0.9 B. 0.8 C. 0.85
 D. 0.5 E. None of the above

**GEORGIA SOUTHERN UNIVERSITY
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2006 VARSITY WRITTEN EXAM ANSWERS**

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|-----|------------------|-----|---------------|
| 1. | D | 21. | A |
| 2. | D | 22. | D |
| 3. | A | 23. | C |
| 4. | E (answer: 3) | 24. | B |
| 5. | D | 25. | B |
| 6. | A | 26. | C |
| 7. | A | 27. | B |
| 8. | C | 28. | A |
| 9. | A | 29. | D |
| 10. | C | 30. | A |
| 11. | A | 31. | D |
| 12. | D | 32. | D |
| 13. | B | 33. | A |
| 14. | D | 34. | B |
| 15. | A | 35. | C |
| 16. | B | 36. | B |
| 17. | A | 37. | C |
| 18. | E (answer: 4680) | 38. | D |
| 19. | D | 39. | D |
| 20. | C | 40. | E (answer: 3) |