

**GEORGIA SOUTHERN UNIVERSITY
INVITATIONAL MATHEMATICS TOURNAMENT
2007 JUNIOR VARSITY WRITTEN EXAM**

Name _____

School _____

Directions:

1. Do not open this test booklet until you are told to do so.
2. Use only a #2 lead pencil.
3. No calculators, slide rules, notes or other aids of any kind may be used.
4. Scratch paper is stapled to the back of the test booklet.
5. This is a 40 question multiple-choice exam. You will be allotted 90 minutes to complete the exam.
6. Geometric figures are not necessarily drawn to scale.
7. Your score will be determined by the formula $40 + 4R - W$ where
R = number of questions answered correctly and W = number of questions answered wrong.
There is no penalty for questions left unanswered.
8. Tie-breakers will be taken from the written exam in order of difficulty. The order will be determined by the number of people that answered each question correctly, with the question(s) correctly answered by the fewest people considered first.

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1. If $f(x) = 4x - \frac{x^2}{4}$, what is $f\left(-\frac{1}{2}\right)$?
- (a) $\frac{-3}{16}$
 - (b) -3
 - (c) $\frac{-33}{16}$
 - (d) -1
 - (e) none of these
2. Find an equation of the line passing through the point $(2,6)$ and perpendicular to the line passing through the points $(1,2)$ and $(3,5)$.
- (a) $2x - 3y = 14$
 - (b) $3x + 2y = 18$
 - (c) $-3x + 2y = 6$
 - (d) $2x + 3y = 22$
 - (e) none of these
3. If $f(x) = x^2 - 3x - 5$, what are the value(s) of k such that $f(k) = k$.
- (a) $-1, 5$
 - (b) 5
 - (c) $-3, 2$
 - (d) $1, -5$
 - (e) none of these

4. Tom has one dollar in 19 coins, some pennies, some nickels, and some dimes. How many **nickels** does he have?
- (a) 5
 - (b) 9
 - (c) 12
 - (d) 16
 - (e) none of these
5. If $d = \frac{c-b}{a-b}$, then $b = ?$
- (a) $\frac{c-d}{a-d}$
 - (b) $\frac{c-ad}{1-d}$
 - (c) $\frac{c+d}{a+d}$
 - (d) $\frac{c+ad}{d-1}$
 - (e) none of these
6. An express train travels 25 miles per hour faster than a freight train. In addition, the express train travels 300 miles in the same time that the freight train travels 200 miles. Find the speed of the **express** train.
- (a) 50 mph
 - (b) 70 mph
 - (c) 75 mph
 - (d) 80 mph
 - (e) none of these
7. Rewrite $\left(x^{\frac{9}{10}}\right)\left(x^{\frac{-1}{5}}\right)$ with a single radical.
- (a) $x^5\sqrt{x}$
 - (b) $\frac{1}{\sqrt[50]{x^9}}$
 - (c) $\sqrt[10]{x^7}$
 - (d) $\sqrt[15]{x^8}$
 - (e) none of these

8. Two arithmetic operations, \square and Δ , are defined as follows:
 $x \square y = (x - y)^2$ and $x \Delta y = x^2 - y^2$, where x and y are real numbers.

Simplify the expression: $\left(\frac{x \square y}{x \Delta y} \right) \square [(1 \square 0) \Delta (x \square x)]$

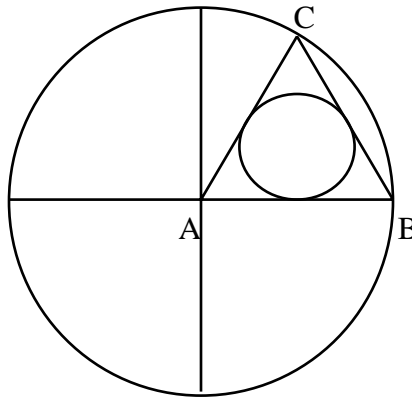
- (a) $\frac{4x^2}{x^2 - y^2}$
(b) 0
(c) $\frac{4x^2 y^2}{x^2 + y^2}$
(d) $\frac{4y^2}{(x + y)^2}$
(e) none of these
9. Find the vertex for the graph of the following function: $f(x) = -2x^2 + 7x - 3$
- (a) $\left(\frac{7}{2}, -3 \right)$
(b) $\left(\frac{7}{4}, \frac{25}{8} \right)$
(c) $\left(\frac{4}{7}, \frac{17}{49} \right)$
(d) $\left(\frac{-7}{4}, \frac{-171}{8} \right)$
(e) none of these
10. If $\frac{1}{4}$ of 2^{30} equals 4^x , then what is the value of x ?
- (a) 12
(b) 14
(c) 15
(d) 26
(e) none of these

11. Simplify the following: $\frac{3a^2b^3c^{-2}}{(a^{-1}b^2c)^3}$

- (a) $\frac{3a^3b}{c^5}$
- (b) $\frac{1}{9ab^{12}c^5}$
- (c) $\frac{b^3c^5}{3a^5}$
- (d) $\frac{3a^5}{b^3c^5}$
- (e) none of these

12. In the picture below, $AC = BC$. C is a point on the large circle. The small circle is inscribed in $\triangle ABC$. Find the area of the small circle (square inches) given that the circumference of the large circle is 6 inches.

- (a) $\frac{3}{4\pi}$
- (b) $\frac{3\pi}{4}$
- (c) $\frac{4}{3\pi}$
- (d) $\frac{4\pi}{3}$
- (e) none of these



13. Simplify the following: $\frac{2 - \frac{1}{2y+1}}{\frac{1}{2y^2-5y-3} - \frac{2}{y-3}}$

- (a) $3 - y$
- (b) 1
- (c) $\frac{4y^2 - 11y - 3}{-4y + 3}$
- (d) $y - 3$
- (e) none of these

14. Simplify the following: $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$

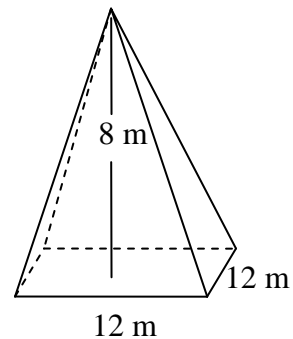
- (a) $2+\sqrt{3}$
- (b) $2-\sqrt{3}$
- (c) $-\frac{1}{\sqrt{5}}$
- (d) 1
- (e) none of these

15. A mathematics class has 15 engineering majors, 12 science majors, and 10 business majors. Five of the engineering majors, 3 of the science majors, and 6 of the business majors are females. Find the probability that a student selected at random is either an engineering major or a female.

- (a) $\frac{24}{37}$
- (b) $\frac{29}{37}$
- (c) $\frac{1}{3}$
- (d) $\frac{5}{37}$
- (e) none of these

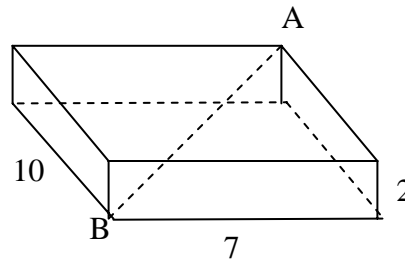
16. The right square pyramid shown has a base that is 12 meters by 12 meters and a height of 8 meters. What is the total surface area, in square meters, of all 5 faces of the pyramid?

- (a) 320.4
- (b) 336
- (c) 384
- (d) 576
- (e) none of these



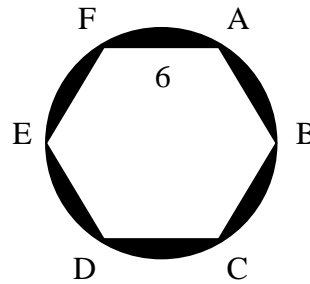
17. For the rectangular solid, find the length of diagonal AB.

- (a) $3\sqrt{17}$
 (b) 19
 (c) $\sqrt{57}$
 (d) $4\sqrt{17}$
 (e) none of these



18. Find the area of the shaded region given that ABCDEF is a regular hexagon with side length 6.

- (a) $12\pi - 9\sqrt{3}$
 (b) $36\pi - 9\sqrt{3}$
 (c) $36\pi - 54\sqrt{3}$
 (d) $27\sqrt{3} - 36\pi$
 (e) none of these



19. A group is arranging a party for which the price per couple will be \$5.00 if 40 or fewer couples attend, but if more than 40 couples attend, the price for all couples will be reduced by \$.10 for each couple above 40. How many couples would yield the maximum revenue?

- (a) 40
 (b) 45
 (c) 50
 (d) 55
 (e) none of these

20. A farmer with 36 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area (square meters) that can be enclosed?

- (a) 81
 (b) 162
 (c) 324
 (d) 1296
 (e) none of these

21. Compare the following: $2 + \sqrt{2}$ \square $2\sqrt{3}$

- (a) $2 + \sqrt{2} > 2\sqrt{3}$
- (b) $2 + \sqrt{2} < 2\sqrt{3}$
- (c) $2 + \sqrt{2} = 2\sqrt{3}$
- (d) $2 + \sqrt{2} \geq 2\sqrt{3}$
- (e) none of these

22. Solve for x : $|x^2 - x - 13| = x + 2$

- (a) $\sqrt{11}, 5$
- (b) $-\sqrt{11}, \sqrt{11}, 5, -3$
- (c) $-\sqrt{11}, \sqrt{11}$
- (d) $5, -3$
- (e) none of these

23. A school has a two-lane running track built around a football field. Each lane is 1 meter wide and each forms semi-circles at the ends of the rectangular field. Determine the distance (meters) between starting positions if two runners must run the same distance to the finish line in one lap around the track.

- (a) 2π
- (b) 2
- (c) π
- (d) 1
- (e) none of these

24. If an arc of 45 degrees on circle A has the same length as an arc of 30 degrees on circle B, find the ratio of the **area** of circle A to the **area** of circle B.

- (a) 2:3
- (b) 3:4
- (c) 4:9
- (d) 5:6
- (e) None of these

25. Container I holds eight red balls and four green balls; containers II and III each hold two red balls and four green balls. A container is selected at random, and a ball is randomly selected from that container. What is the probability that the ball selected is green?
- (a) $\frac{4}{9}$
 - (b) $\frac{5}{9}$
 - (c) $\frac{4}{729}$
 - (d) $\frac{5}{6}$
 - (e) none of these
26. The midpoints of the three sides of a triangle are as follows: $L(2,1)$ $M(5,2)$ $N(1,4)$. Find the coordinates of the vertices of the triangle.
- (a) $(-2,3), (4,5), (6,-1)$
 - (b) $(-2,3), (2,5), (12,9)$
 - (c) $(1,2), (3,3), (8,5)$
 - (d) $(-5,2), (9,0), (1,4)$
 - (e) none of these
27. Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?
- (a) 5
 - (b) 10
 - (c) 20
 - (d) 30
 - (e) none of these
28. How many integers between 100 and 1000 are multiples of 7?
- (a) 127
 - (b) 128
 - (c) 129
 - (d) 130
 - (e) none of these

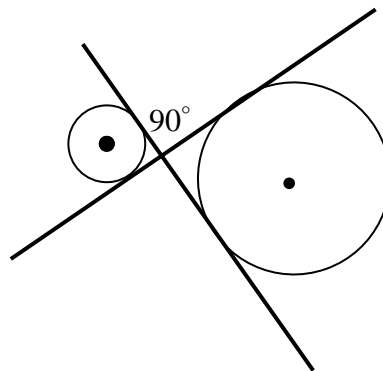
29. A used-car dealer sold two cars and received \$560 for each car. One of these transactions resulted in a 40 percent profit for the dealer, whereas the other resulted in a 20 percent loss. What is the dealer's net profit on the two transactions?

- (a) \$ 20
- (b) \$ 80
- (c) \$ 112
- (d) \$ 224
- (e) none of these

30. A box contains 1001 red marbles and 1001 black marbles. Let P_S be the probability that two marbles drawn at random from the box are the same color, and let P_D be the probability that they are different colors. Find $|P_D - P_S|$.

- (a) $\frac{2000}{2001}$
- (b) $\frac{1999}{2001}$
- (c) 0
- (d) $\frac{1}{2001}$
- (e) none of these

31. In the diagram, the tangents to the two circles intersect at 90 degree angles, as shown. If the radius of the smaller circle is 2 and the radius of the larger circle is 5, what is the distance between the centers of the two circles?

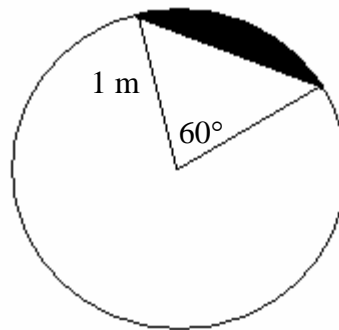


- (a) $7\sqrt{2}$
- (b) $10\sqrt{2}$
- (c) $5\sqrt{3}$
- (d) 10
- (e) none of these

32. The arithmetic mean of three numbers is 82. If the mean of the first two numbers is 79, what is the third number?
- (a) 86
 - (b) 88
 - (c) 90
 - (d) 92
 - (e) none of these

33. Find the exact area (square meters) of the shaded region.

- (a) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$
- (b) $30\pi - \frac{\sqrt{3}}{4}$
- (c) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
- (d) $30\pi - \frac{\sqrt{3}}{8}$
- (e) none of these



34. There are 40 students in a class. Blue is the favorite color of 22 students; 19 own cars; 13 like flowers; blue is the favorite color of 10 students who own cars; blue is the favorite color of 7 students who like flowers; 8 own cars and like flowers; and blue is the favorite color of 5 students who own cars and like flowers. How many students do not like flowers, do not like the color blue, and do not own a car?
- (a) 2
 - (b) 4
 - (c) 6
 - (d) 8
 - (e) none of these

35. The diagonal of one square is four times the length of the diagonal of another. How many times larger is the area of the larger square than the smaller?
- (a) 4
 - (b) 8
 - (c) 16
 - (d) 64
 - (e) none of these
36. If $x + \frac{1}{x} = 4$, then what is the value of $x^3 + \frac{1}{x^3}$?
- (a) 12
 - (b) 52
 - (c) 64
 - (d) $30\sqrt{3}$
 - (e) none of these
37. In a three-digit number, the hundreds digit is greater than 5, the tens digit is greater than 4 but less than 8, and the units digit is the smallest prime number. How many three-digit numbers satisfy all these conditions?
- (a) 6
 - (b) 9
 - (c) 12
 - (d) 24
 - (e) none of these

38. Two complementary angles, A and B, have measures in the ratio of 7 to 23, respectively. What is the ratio of the measure of the supplement of angle A to the measure of the supplement to angle B?

- (a) $\frac{53}{37}$
- (b) $\frac{23}{7}$
- (c) $\frac{7}{23}$
- (d) $\frac{37}{53}$
- (e) none of these

39. The figure to the right shows an example of a 4-digit customer identification code. If the digits in the code $\{0, 1, 2, \dots, 9\}$ must appear in descending order and no digit can be used more than once, what is the difference between the largest and smallest possible codes?

Example of Customer Identification Code

7	5	2	1
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- (a) 6,666
- (b) 5,555
- (c) 5432
- (d) 4444
- (e) none of these

40. Paco uses a spinner to select a number from 1 through 5, each of which has equal probability of being selected. Manu uses a different spinner to select a number from 1 through 10, each of which has equal probability. What is the probability that the product of Manu's number and Paco's number is less than 30?

- (a) $\frac{43}{49}$
- (b) $\frac{9}{50}$
- (c) $\frac{41}{49}$
- (d) $\frac{41}{50}$
- (e) none of these

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SOLUTIONS

- | | | | |
|-----|---|-----|---|
| 1. | C | 21. | B |
| 2. | D | 22. | A |
| 3. | A | 23. | A |
| 4. | B | 24. | C |
| 5. | B | 25. | B |
| 6. | C | 26. | A |
| 7. | C | 27. | B |
| 8. | D | 28. | B |
| 9. | B | 29. | A |
| 10. | B | 30. | D |
| 11. | D | 31. | A |
| 12. | A | 32. | B |
| 13. | A | 33. | A |
| 14. | B | 34. | C |
| 15. | A | 35. | C |
| 16. | C | 36. | B |
| 17. | A | 37. | C |
| 18. | C | 38. | A |
| 19. | B | 39. | A |
| 20. | B | 40. | D |