Applications of the Fourier Transform in Imaging
Highpass and Lowpass Filters

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Introduction

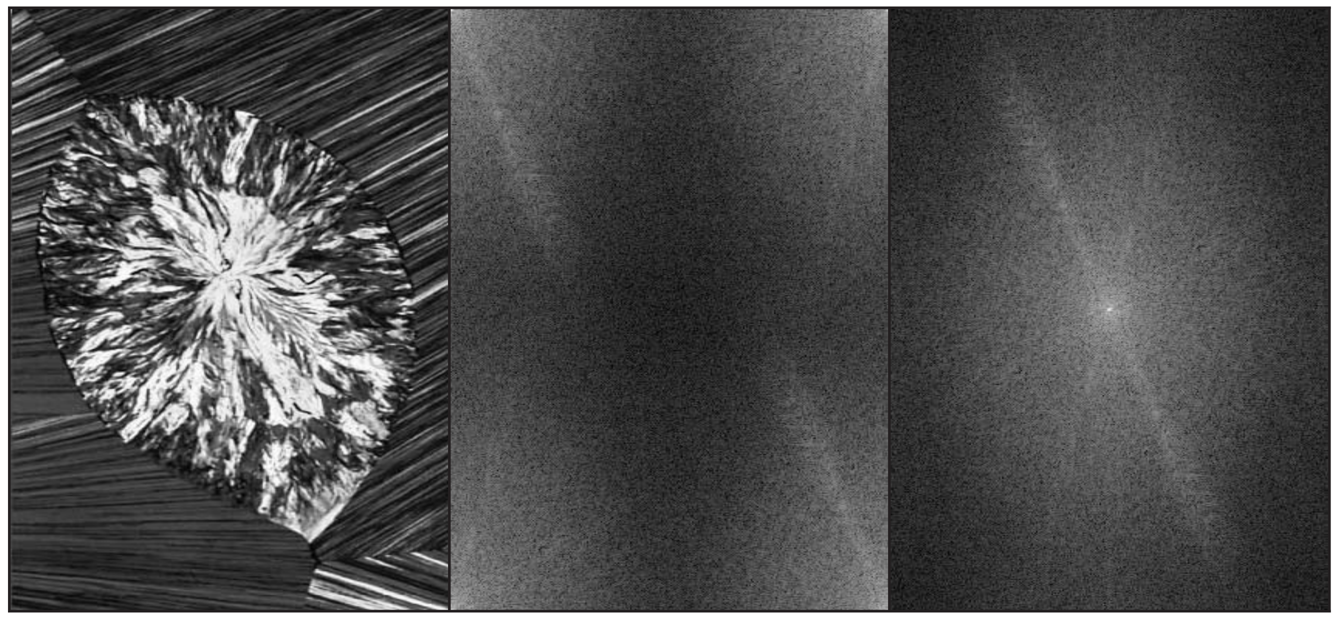
Filtering in the frequency domain has advantages over filtering in the spatial domain. Sharpening and blurring filters, building blocks for certain image enhancement techniques, are easily understood in the frequency domain. We present techniques for the filtering process and design of lowpass and highpass filters. The following was preliminary research into adaptive filter design in radar imaging, supported by **NSF-DMS-REU-06553331**

2D Discrete Fourier Transform

We denote images by f(x,y). The 2D Discrete Fourier Transform is given by

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-i2\pi(\frac{ux}{M}+\frac{vy}{n})}$$

for an Image of size M x N. Throughout this presentation, we will center the DFT by first using Matlab's fftshift() command for purposes that will become apparent during the filter design.



(1) Image of a Crystal (2) Uncentered FT of image (3)

The centered FT images contain two pieces of information that we need to design high-pass and lowpass filters:
(1) Low frequencies, the frequencies representing the general structure of the image, appear around the center of the transform.
(2) High frequencies, frequencies representing sharp transitions, edges, and noize, appear furthur away from the center.

Fourier Filtering Process

The process of filtering in the frequency domain is simple and easy to implement.

(1) Compute the DFT
(2) Center the transform
(3) Multiply by a filter function pointwise on the Fourier transform of the image
(4) Compute the Inverse Fourier transform
(5) Uncenter the transform
(6) Obtain the real part of the Inverse Fourier transform

Gaussian Lowpass and Highpass Filters

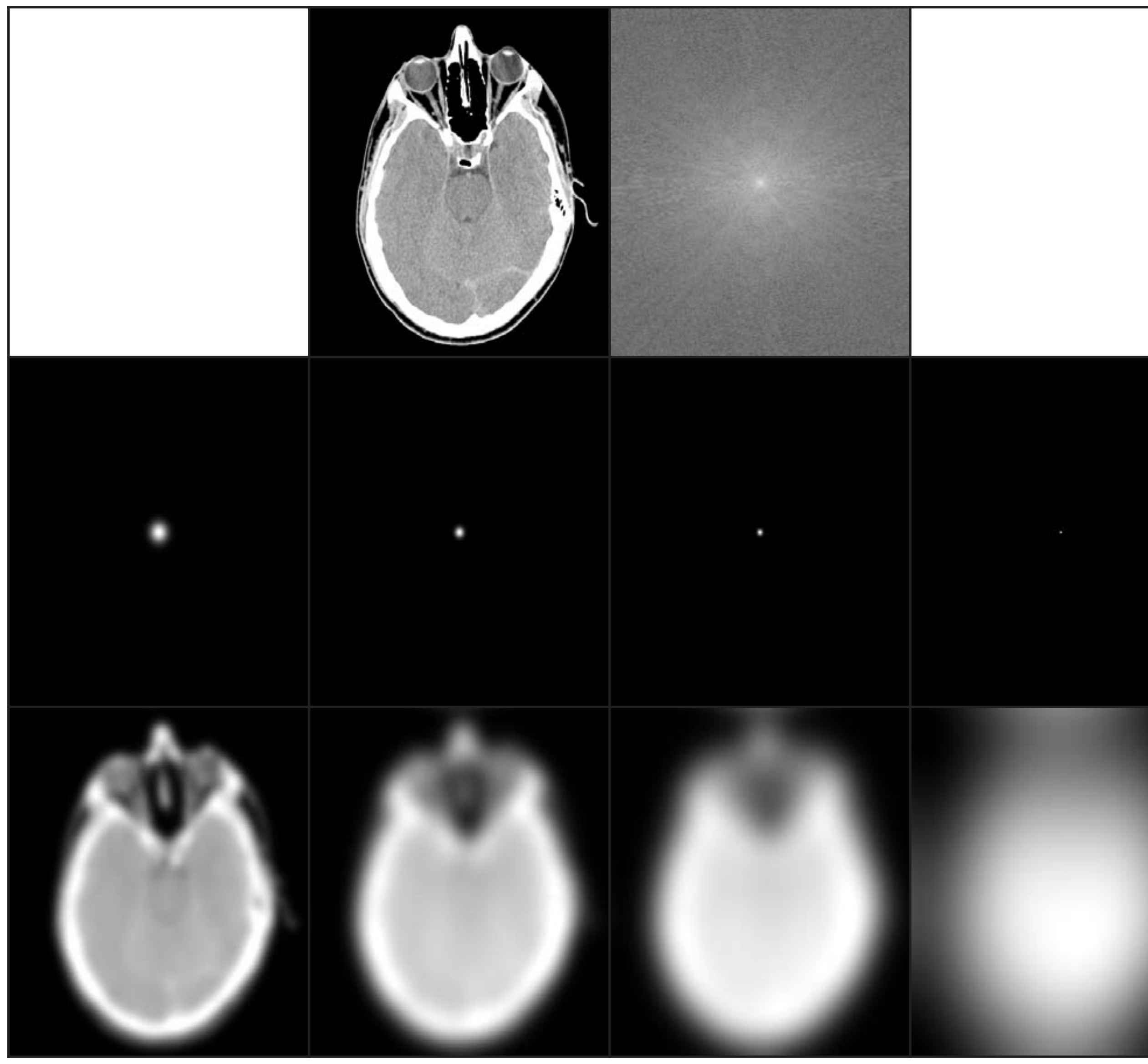
The 2D form of the Gaussian function is given by

$$Filter = e^{\frac{-D^2(u,v)}{2\sigma^2}}$$

Where

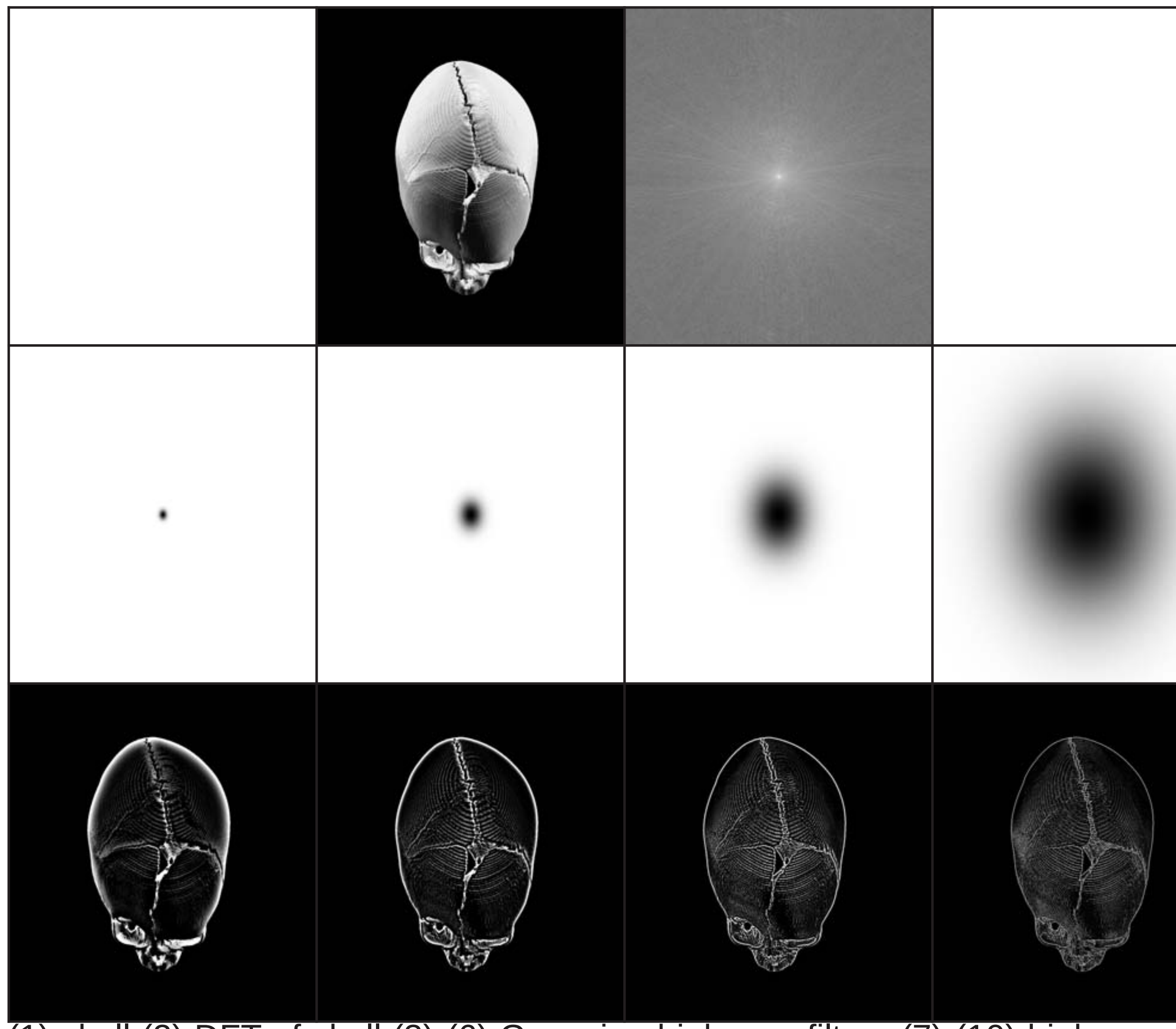
$$D(u,v) = [(u-\frac{M}{2})^2 + (v-\frac{N}{2})$$

The Filter above is a lowpass filter. That is, it deletes all high frequencies from the transform and leaves the low frequencies intact.



(1) MRI data (2) MRI DFT (3)-(6) Lowpass Filters (7)-(10)

We can obtain a Gaussian highpass filter by subtracting the lowpass filter by 1.



(1) skull (2) DFT of skull (3)-(6) Gaussian highpass filters (7)-(10) highpass tered images

Reference: Digital Image Processing - Gonzalez and Woods